1 (10 points) Set up and solve the baseline (centralized) model of Chapter 2. Solve using general parameter values for $T, A_G, \alpha,$and γ. Fix the capital stock at $k = 1$.

Utility is log linear in consumption, c, and leisure, x. The explicit form is $U(c, x) = \ln(c) + \alpha \ln(x)$.

The production function is Cobb-Douglas and is given as $c = f(l) = A_G l^\gamma$

Time available, T, is just equal to labor plus leisure, $T = l + x$.

a. **Show the expression for the** MRS_{xc}

$MRS_{xc} = \frac{MU_x}{MUc} = \frac{\frac{\alpha}{x}}{\frac{1}{c}}$

b. **Show that the** MP_l $=MRS_{xc}$.

Modify the Utility function by substituting for c and x with $A_G l^\gamma$and $T - l$. Take first order conditions and calculate the marginal rate of substitution between leisure and consumption.

Taking the derivative with respect to l, by using the "chain rule" of calculus, gives that

$$\frac{\partial u}{\partial c}\frac{\partial f(l)}{\partial l} + \frac{\partial u(c,x)}{\partial x}\frac{\partial (T-l)}{\partial l} = 0.$$

Rearranging this gives the basic characterizing equilibrium condition of the economy: MP_l $=MRS_{xc}$

$$MP_l \equiv \frac{\partial f(l)}{\partial l} = \frac{\frac{\partial u(c,x)}{\partial x}}{\frac{\partial u(c,x)}{\partial c}} \equiv MRS_{x,c}.$$

c. **Show that the labor supply does not depend on the level of technology.**

$$\underset{l}{\text{Max}}\ u[f(l), T-l] = \ln(A_G l^\gamma) + \alpha \ln(T-l).$$

$$\begin{aligned} \frac{\partial u[f(l), T-l]}{\partial l} &= \frac{\partial[\ln(A_G l^\gamma) + \alpha\ln(T-l)]}{\partial l} = 0; \\ 0 &= \frac{\gamma A_G}{A_G l} - \alpha\left(\frac{1}{T-l}\right). \end{aligned}$$

The solution for the labor l can be found since this equilibrium condition is just one equation in the unknown l:. The solutions shows that labor depends on T, but does not depend on A_G.

$$\begin{aligned} \gamma\ (T-l) &= \alpha l, \\ l &= \frac{T\gamma}{\alpha+\gamma}. \end{aligned}$$

d. **What happens to utility, output, and labor if the household has more total time available to be used for work or play? Explain why in each case.**

If time goes up, none of the equilibrium conditions change. The household allocates the additional time in the same ratio as it allocated T originally. In this case, utility, output and labor all rise, as does leisure.

2 (10 points) What is a small open economy?

The small refers to the fact that the people in the country take the interest rate as given, determined by the rest of the world. Open refers to the fact that it can trade with the rest of the world.

a. **Set up the model of the small open endowment economy of Chapter 7.**

c_0 consumption in current period, c_1 consumption in next period

Income endowment y_0 and y_1; r real interest rate.

β discount factor for future income, $\beta \in (0, 1)$.

utility:

$$u(c_0, c_1) \equiv \ln c_0 + \beta \ln c_1,$$

Budget (Wealth) Constraint W

$$W \equiv y_0 + \frac{y_1}{1+r} = c_0 + \frac{c_1}{1+r}.$$

b. **What four factors determine whether a small country will be a lender or a borrower?**

The factors are the relative sizes of $y_0, y_{1,}r$,and ρ where $\beta = \frac{1}{1+\rho}$. The agent wants to smooth consumption across both periods and will borrow future endowment or lend current endowment at the world interest rate until the $\ln c_0 + \alpha \ln c_1$ reaches its maximum value.

3 (10 points) Set up the decentralized model of Chapter 8. Write down the Bellman Equation, and the four contraints.

a.

$$V(k_t) = \underset{c_t, x_t, l_t, k_{t+1}}{Max} : u(c_t, x_t) + \beta V(k_{t+1}),$$

$$\text{where } u(c_t, x_t) = \ln c_t + \alpha \ln x_t.$$

$$y_t = A_G l_t^{\gamma} k_t^{1-\gamma},$$

$$y_t = c_t + i_t.$$

$$i_t = k_{t+1} - k_t(1 - \delta_k)$$

$$T = x_t + l_t.$$

a. **Write down the modified Bellman equation with constraints substituted in for consumption and leisure.**

$$V(k_t^s) = \underset{l_t^s, k_{t+1}^s}{Max} : u\left[w_t l_t^s + r_t k_t^s + \Pi_t - k_{t+1}^s + k_t^s(1 - \delta_k), T - l_t\right] + \beta V\left(k_{t+1}^s\right).$$

$$\underset{l_t^d, k_t^d}{Max}\, \Pi_t = A_G \left(l_t^d\right)^{\gamma} \left(k_t^d\right)^{1-\gamma} - w_t l_t^d - r_t k_t^d.$$

b. **Write down the first order conditions and the envelope condition. Substitute in the envelope condition to get the two FOCs that represent the intertemporal and intratemporal margins.**

$$\begin{aligned}
0 &= \frac{\partial u\left(c_t^d, x_t\right)}{\partial c_t} w_t - \frac{\partial u\left(c_t^d, x_t\right)}{\partial x_t}; \\
0 &= \frac{-\partial u\left(c_t^d, x_t\right)}{\partial c_t^d} + \beta \frac{\partial V\left(k_{t+1}^s\right)}{\partial k_{t+1}^s}. \\
Envelope &: \frac{\partial V\left(k_t^s\right)}{\partial k_t^s} = \frac{\partial u\left(c_t^d, x_t\right)}{\partial c_t^d}(1 + r_t - \delta_k). \\
&\Rightarrow \frac{c_{t+1}^d}{c_t^d} = \frac{1 + r_{t+1} - \delta_k}{1 + \rho}, \text{Intertemporal Margin} \\
w_t &= \frac{\frac{\alpha}{x_t}}{\frac{1}{c_t^d}}.\text{Intratemporat Margin}
\end{aligned}$$

4 (10 points) Understanding how to use the GE framework in 2 dimensions.

a. **Show how AS-AS curves change when technology (A_G) rises.**

See Figure 8.9 on page 354 of text.

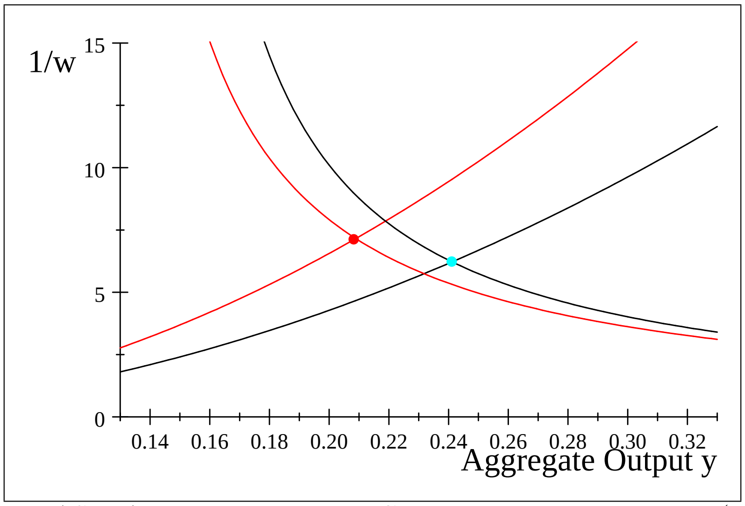
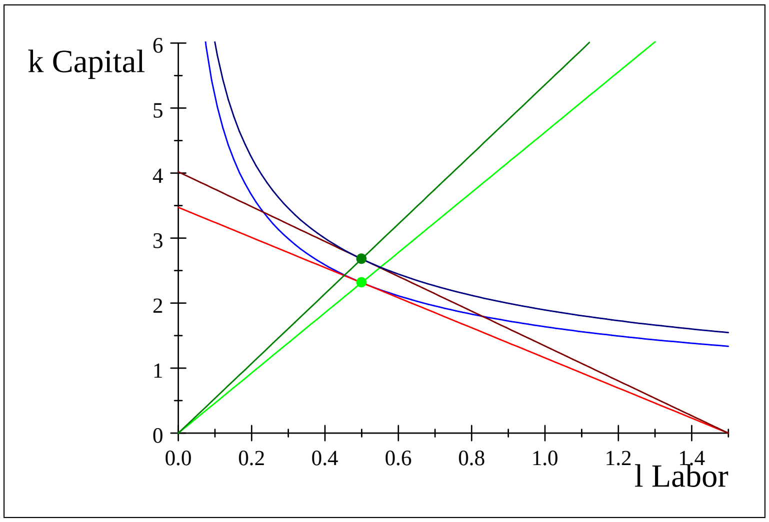


Figure 8.9. $AS - AD$ Equilibrium with Goods Productivity Increase (in black) in Example 8.2, compared to the Baseline (in red) of Example 8.1.

b. **Show how the factor market equilibrium changes wih technology rises.**

See Figure 8.11 on page 357

Isoquant Shift Up, Isocost Pivots Up, Input Ratio Up



Example 8.11. Factor Market Equilibrium with Goods Productivity Increase of Example 8.2.

c. **Show how AS-AS curves change when the time endowment increases.**

See Figure 8.13 on page 361

AS-AD Shift Out, Wage the Same

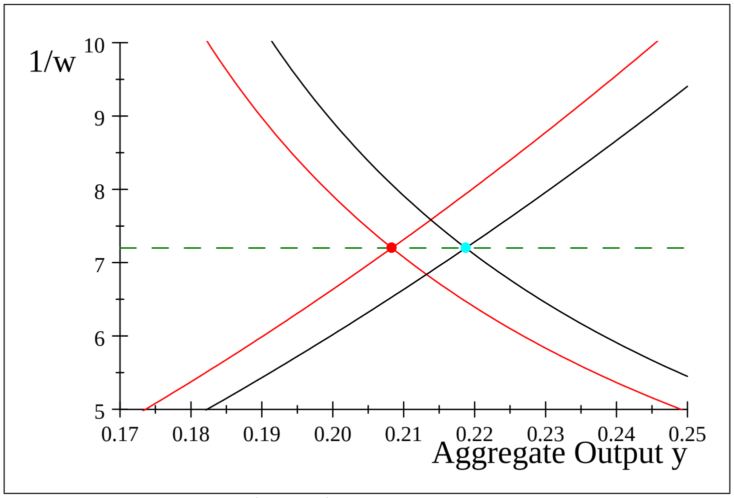
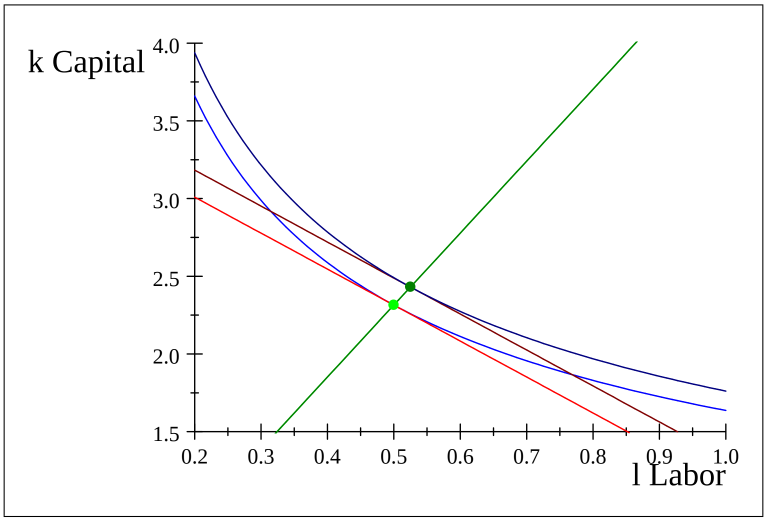


Figure 8.13. Shift in $AS - AD$ with Time Endowment Increase

d. **Show how the Factor Market Equilibrium changes when the time endowment increases.**

See Figure 8.15 on page 364

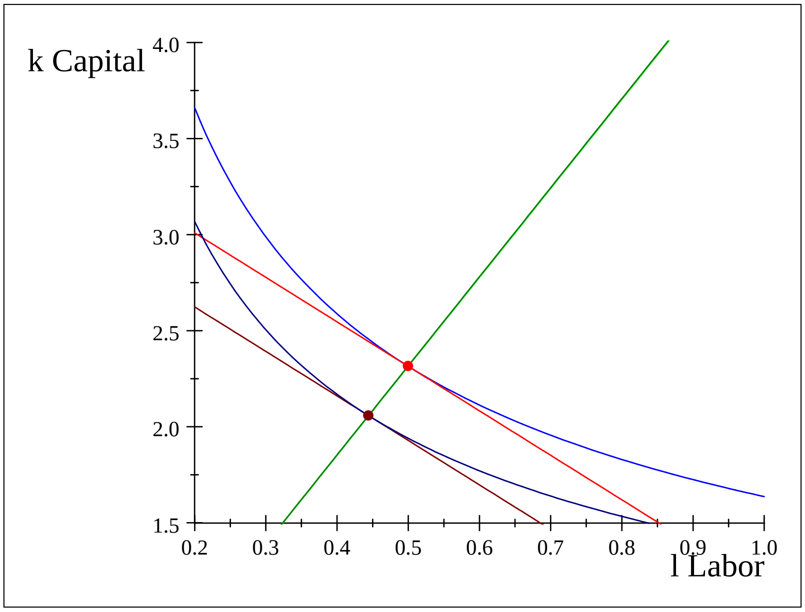
Employment Rises, Factor Ratio Unchanged



Example 8.15. Factor Market Equilibrium with Time Endowment Increase of Example 8.3.

e. **Show how the Factor Market equilibrium changes when a labor income tax is introduced.**

See Figure 9.15 on page 402



Example 9.15. Factor Market Equilibrium with Labor Income Tax τ_l, in Example 9.7 (lower) and Baseline (upper).

5 (10 points) Please explain each of the following underlined terms and answer any questions (each answer is worth 2 points).

a. **Consumption smoothing** is the economic concept used to express the desire of people to have a stable path of consumption. It refers to the behavior of agents who are maximizing expected lifetime utility. Depending on relative prices, time preference, and expectations of future income, the agent will change the amount of time spent working or trade in asset markets to make consumption relatively smooth over time.

b.**Intratemporal Margin (in the example of question 3, which first order condition defines this margin for this model?**) The intratemporal margin is the margin of tradeoff between consumption and leisure within a given period. In the model of question 3 (Chapter 8), this is the first order condition that comes from the derivative of Utility with respect to labor.

c. **Define the Rate of time preference.** This is the internal discount rate that an agent (household) uses to discount the utility of future income. In neoclassical economics, the rate of time preference is usually taken as a parameter in an individual's utility function which captures the trade off between consumption today and consumption in the future, and is thus exogenous and subjective. It is also the underlying determinant of the real rate of interest.

d. **What use is the Envelope Condition?** The envelope condition refers to the fact that the derivative of a value function with respect to a state variable exists if the underlying period-by-period function has a continuous derivative with respect to the state variable. It provides an extra equilibrium condition that may be needed to solve a Bellman equation.

e. **Why do we impose the Transversality Condition?** The transversality condition for an infinite horizon dynamic optimization problem is the boundary condition determining a solution to the problem's first-order conditions together with the initial condition. The transversality condition requires the present value of the value function at t+1 to converge to zero as the planning horizon goes to infinity. In a model with asset trading, the transversality conditon says that the household cannot be expected to carry debt into infinity. Without the transversality condition, utility may rise in an unbounded way.