Introduction to Chapter 8 Dynamic Analysis and *AS*-*AD* (pp 322-371)

March 13, 2017

Homework for March 13

- Read through the end of Chapter 8
- Replicate Figures 8.4, 8.7, and 8.8. (For the exam you will have to understand how these three figures change when there is a change in productivity or the time endowment.)
- Continue to turn in old quizzes. Feel free to cooperate in homework, but be sure that you have the ability to answer questions on you own by March 29.

Derive the AD/AS equations for plotting

- Baseline calibration
- $\gamma = 1/3$
- $\alpha = 0.5$
- $T = 1$
- $\rho = 0.03$
- $A_G = 0.15$
- $\delta_k = 0.03$
- Given $k_t = 2.3148$
- With no growth, $r = 0.06$

Consumption Demand

$$x_t = \frac{\alpha c_t^d}{w_t},\ x_t = T - l_t, \Rightarrow l_t^s = T - \frac{\alpha c_t^d}{w_t}.$$

$$c_t^d = w_t\left(T - \frac{\alpha c_t^d}{w_t}\right) + r_t k_t + \Pi_t - k_{t+1} + k_t(1 - \delta_k).$$

$$c_t^d = \frac{w_t T + r_t k_t + \Pi_t - k_{t+1} + k_t(1 - \delta_k)}{1 + \alpha},$$

$$c_t^d = \frac{w_t T + \Pi_t + k_t\left(1 + r_t - \delta_k - \frac{k_{t+1}}{k_t}\right)}{1 + \alpha}.$$

Aggregate Demand

$$i_t = k_{t+1}^d - k_t^d(1-\delta_k),$$

$$k_{t+1}^d = k_t^d,$$

$$i_t = k_t^d - k_t^d(1-\delta_k) = \delta_k k_t^d.$$

$$k_t^s = k_t^d = k_t,$$

$$AD\ :\ y_t^d = c_t^d + i_t = \left(\frac{1}{1+\alpha}[w_t T + \rho k_t]\right) + \delta_k k_t,$$

$$y_t^d = \frac{w_t T + k_t[\rho + (1+\alpha)\delta_k]}{1+\alpha}.$$

$$Inversely\ :\ \frac{1}{w_t} = \frac{T}{y_t^d(1+\alpha) - k_t[\rho + (1+\alpha)\delta_k]}.$$

Aggregate Supply

$$l_t^d = \left(\frac{\gamma A_G}{w_t}\right)^{\frac{1}{1-\gamma}} k_t.$$

$$y_t^s = A_G\left(l_t^d\right)^{\gamma}(k_t)^{1-\gamma}.$$

$$AS : y_t^s = A_G(k_t)^{1-\gamma}\left(\frac{\gamma A_G}{w_t}\right)^{\frac{\gamma}{1-\gamma}}(k_t)^{\gamma} = A_G^{\frac{1}{1-\gamma}}\left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t.$$

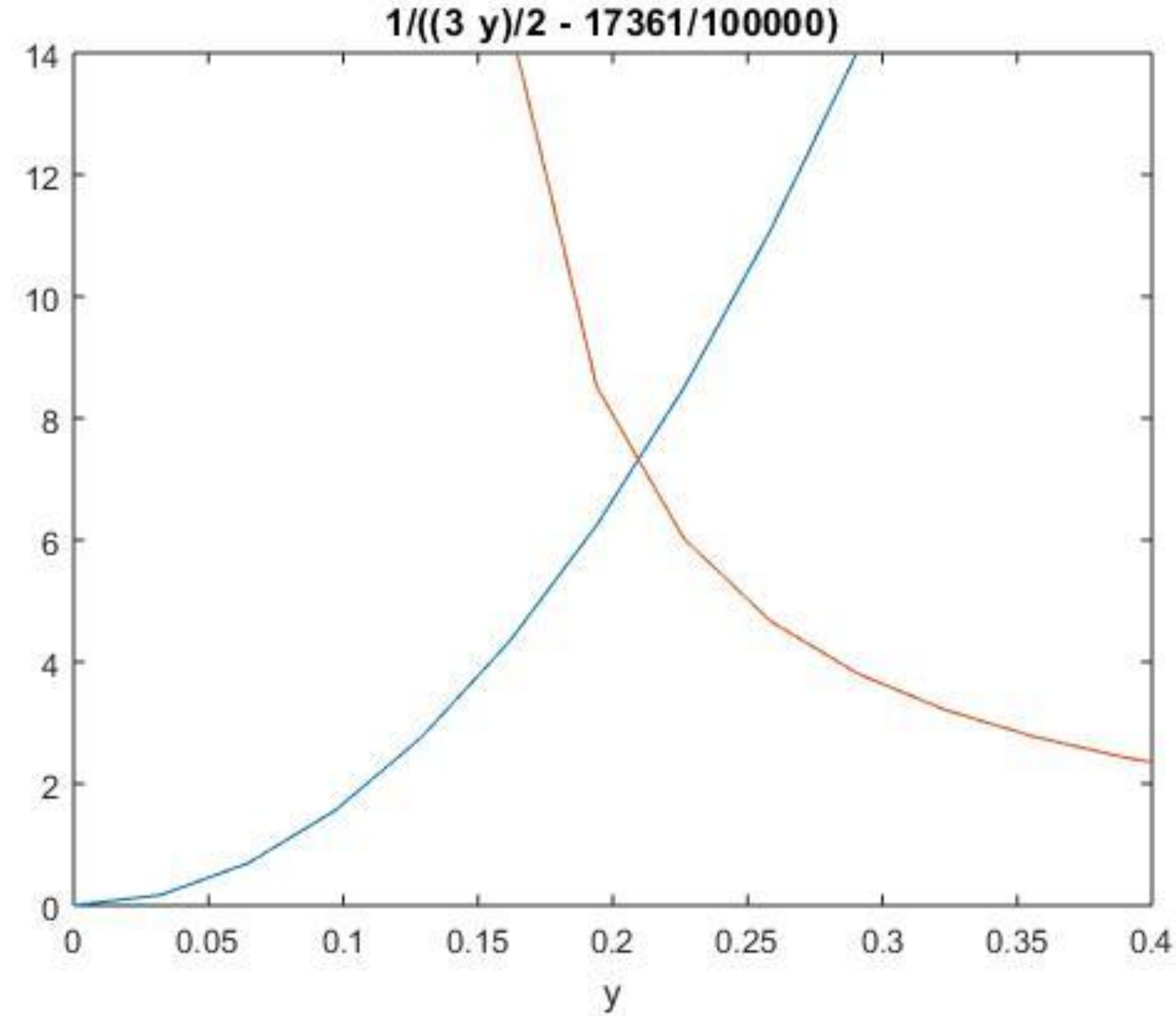
$$Inversely : \frac{1}{w_t} = \frac{(y_t^s)^{\frac{1-\gamma}{\gamma}}}{\gamma A_G^{\frac{1}{\gamma}}(k_t)^{\frac{1-\gamma}{\gamma}}}.$$

Matlab Program to Replicate Figure 8.4

```
% Model_Gillman_ch8.m
% This model includes log utility and cobb douglas production
% This program was last updated March 7, 2017
clc
clear
%****************************************
% Calibrated Parameters
%****************************************
parameter_names = ['gamma',' ','delta',' ', 'rho',' ', 'Ag',' ', 'alpha',' ',
'time' ];
gam = 1/3 ;     % labor's share of output
delt = 0.03;    % steady state capital depreciation rate
rho = 0.03;     % rate of time preference
ag = 0.15;      % Technology Factor, also called productivity factor
alph = 0.5;     % Weight on leisure in log utility
tim = 1;        % Time available for work and leisure
```

```
parameter_values = [gam delt rho ag alph tim];
rk = rho + delt ;
k = 2.3148 ;
%*****************************
% output
%*****************************
 %********** AD-AS ***Figure 8.4**************
syms y
f(y) = (y^((1-gam)/gam)/((gam*ag^(1/gam))*k^((1-gam)/gam))); % AS eq 8.62
g(y) = tim/(y*(1+alph)-k*(rho+(1+alph)*delt)) ;                % AD eq 8.57
LIMS = [0 .4 0 14];
ezplot(f,LIMS)
hold
ezplot(g,LIMS)
```

Figure 8.4



Chapter 8 takes the solution of *k* as given.
Chapter 10 shows ways to compute *k*

$$k_t = \frac{T\gamma A_G^{\frac{1}{\gamma}}\left[\frac{(1-\gamma)}{\rho+\delta_k}\right]^{\frac{1-\gamma}{\gamma}}}{(\gamma+\alpha)\left(\frac{\rho+\delta_k}{(1-\gamma)}\right)-\alpha\delta_k}$$

All Other Variables are a Function of k and parameters

$$r_t = \rho + \delta_k;$$

$$l_t = \left[\frac{\rho + \delta_k}{(1-\gamma)A_G} \right]^{\frac{1}{\gamma}} k_t;$$

$$w_t = \gamma A_G \left[\frac{(1-\gamma)A_G}{\rho + \delta_k} \right]^{\frac{1-\gamma}{\gamma}};$$

$$y_t = A_G \left[\frac{\rho + \delta_k}{(1-\gamma)A_G} \right] k_t;$$

$$i_t = \delta_k k_t;$$

$$c_t = A_G \left[\frac{\rho + \delta_k}{(1-\gamma)A_G} \right] k_t - \delta_k k_t.$$

$$x_t = T - l = T - \left[\frac{\rho + \delta_k}{(1-\gamma)A_G} \right]^{\frac{1}{\gamma}} k_t;$$

Matlab Code

Replace k=2.3148 with the formula from chapter 10

```
knum = (tim*gam*ag^(1/gam))*((1-gam)/
                    (rho+delt))^((1-gam)/gam) ;
kdenom = (gam+alph)*((rho+delt)/
                    (1-gam))-alph*delt ;
k = knum/kdenom ;
```

Figure 8.4

- %*********** AD-AS ***Figure 8.4***************
- syms y
- f(y) = (y^((1-gam)/gam)/((gam*ag^(1/gam))*k^((1-gam)/gam))); % AS eq 8.62
- g(y) = tim/(y*(1+alph)-k*(rho+(1+alph)*delt)) ;   % AD eq 8.57
- LIMS = [0 .4 0 14];
- ezplot(f,LIMS)
- hold
- ezplot(g,LIMS)

Figure 8.7

```
%****** Factor Market Equilibrium *****Figure 8.7 ***
syms l
f(l) = (1/rk)*(y-w*l);                          % Isocost eq 8.72
g(l) = (y/(ag*l^gam))^(1/(1-gam));    % Isoquant eq 8.73
h(l) = l/(((rho+delt)/((1-gam)*ag))^(1/gam));
                                                % Input ratio eq 8.74
LIMS = [0.0 1.4 0 6];
 ezplot(f,LIMS)
hold
ezplot(g,LIMS)
ezplot(h,LIMS)
```

Figure 8.8

- %****** General Equilibrium *****Figure 8.8 ********
- %*******consumption on vertical axis****************
- syms l
- f(l) = ag*(l^gam)*k^(1-gam)-delt*k;   % Production eq 8.75
- g(l) = exp(u)/((tim-l)^alph);   % Utility eq 8.76
- h(l) = w*l+rho*k;   % Budget line eq 8.77
- LIMS = [0.0 1.0 0.00 0.25];
- ezplot(f,LIMS)
- hold
- ezplot(g,LIMS)
- ezplot(h,LIMS)

Homework for March 15

- Continue to turn in old quizzes.
- Replicate figures 8.9, 8.11, and 8.12
- Replicate figures 8.13, 8.15, and 8.16
- Due by March 22
- Create new figures that combine the productivity and the time endowment increase together.
  - AD-AS, Factor market equilibrium and time endowment and productivity
- Read Chapter 9 – labor tax is most important
- Replicate figure 9.15