Chapters 18: Public Finance (pp. 766-791)

May 3, 2017

Homework for May 3

- Read Chapter 18. Public Finance pages 766-791
- Quiz 23

1. 1. Set up and solve the model with bonds (Sections 17.2.1 and 17.2.2).
2. Write out the government's wealth constraint.
3. What is Ricardian Equivalence?
4. What is the transversality condition that underlies Ricardian equivalence?
5. Explain the advantages and disadvantages of printing money rather than raising taxes to pay for government spending.

Extending the model with bonds

- How does the model change if we allow inflation?
- Can we add a nominal price and inflation without adding money?
  - Money is just a unit of account.
  - Short-term bonds are used as money.
  - All trades are electronic based on deposits of bonds.
- Before going to the monetary model, discuss the role of public finance.
- Suppose we consider 'starting up a government' in the Garden of Eden.

The Government Budget Constraint

Creation Day 8: $B_0 + M_0 = G_{-1}$

Creation Day 9: $G_0 + R_0 B_0 =$

$$(\tau_l w_0 l_0 h_0 + \tau_k r_0 k_0 + \tau_c c_0) + (B_1 - B_0) + (M_1 - M_0).$$

Creation Day 10 and forever after: $G_t + R_t B_t =$

$$(\tau_l w_t l_t h_t + \tau_k r_t k_t + \tau_c c_t) + (B_{t+1} - B_t) + (M_{t+1} - M_t).$$

Government Wealth Constraint

$$G_{-1} + \frac{G_0}{1+R_0} + \frac{G_1}{(1+R_0)(1+R_1)} + \ldots$$

$$= \frac{\tau_l w_0 l_0 h_0 + \tau_k r_0 k_0 + \tau_c c_0}{1+R_0} + \frac{\tau_l w_1 l_1 h_1 + \tau_k r_1 k_1 + \tau_c c_1}{(1+R_0)(1+R_1)} + \ldots$$

$$+ B_0 - \frac{B_0}{1+R_0} - \frac{R_0 B_0}{1+R_0} + \frac{B_1}{1+R_0} + \frac{B_2 - B_1 - B_1 R_1}{(1+R_0)(1+R_1)} + \ldots$$

$$+ M_0 + \frac{M_1 - M_0}{1+R_0} + \frac{M_2 - M_1}{(1+R_0)(1+R_1)} + \ldots$$

The present value of government spending is equal to the sum of the present value of tax revenue, plus the present value of debt, plus the present value of money creation.

Show that the present value of debt must be zero!

Present value of debt must be zero

$$B_0 - \frac{B_0}{1+R_0} - \frac{R_0 B_0}{1+R_0} = B_0 - \frac{B_0(1+R_0)}{1+R_0} = B_0 - B_0 = 0.$$

$$\frac{B_1}{1+R_0} - \frac{B_1 + B_1 R_1}{(1+R_0)(1+R_1)}$$

$$= \frac{B_1}{1+R_0} - \frac{B_1(1+R_1)}{(1+R_0)(1+R_1)} = \frac{B_1}{1+R_0} - \frac{B_1}{1+R_0} = 0.$$

$$\lim_{j\to\infty}\left[\frac{B_{t+j}}{(1+R_{t+1})(1+R_{t+2})\cdots(1+R_{t+j})}\right] = 0.$$

Ricardian Equivalence

- Ricardian equivalence: all spending eventually paid for by taxes.
- Spending without raising taxes (forever) violates the transversality condition.
- With the transversality condition the wealth constraint: The discounted stream of government expenditure equals discounted streams of taxes and money creation.

Issues in Ricardian Equivalence

- Are bonds net wealth?
- Can there be temporary deviations from Ricardian Equivalence?
  - Does this question make any sense to you?
  - Can this generation take advantage of future generations?
- What is the concept of “credibility” for the government’s financial authority?

Balanced growth

- Assume the transversality condition holds so present value of debt is zero.
- Look at the present value of taxes, spending and money creation.
- Simplify let tax revenue be τy_0 in period 0, growth and all interest rates are constant.
- Show the present value of taxes:

Balanced growth

- Assume the transversality condition holds so present value of debt is zero.
- Look at the present value of taxes, spending and money creation.
- Simplify let tax revenue be τy_0 in period 0, growth and all interest rates are constant. Then

$$\frac{\tau y_0}{1+R_0} + \frac{\tau y_0(1+g)}{(1+R_0)(1+R_0)} + \frac{\tau y_0(1+g)^2}{(1+R_0)(1+R_0)(1+R_0)} + \ldots$$

$$= \frac{\tau y_0}{1+R_0}\left(1 + \frac{1+g}{1+R_0} + \left(\frac{1+g}{1+R_0}\right)^2 + \ldots\right)$$

$$= \frac{(\tau y_0)}{R_0 - g} = \frac{\tau y}{\rho(1+g)}.$$

Remember from Chapter 11 and Chapter 17

With growth, but no bonds

$$1+g = \frac{(c_{t+1})^d}{(c_t)^d} = \frac{1+r_t-\delta_k}{1+\rho},$$

$$r_t - \delta_k = (1+g)(1+\rho) - 1,$$

$$r_t - \delta_k - g = (1+g)(1+\rho) - 1 - g = \rho(1+g).$$

With no growth define *R* in Chapter 17

$$\frac{c_t}{\beta c_{t-1}} = 1+R_t, \;\; \beta \equiv \frac{1}{1+\rho}, \;\; \frac{c_t}{c_{t-1}} = \frac{1+R_t}{1+\rho};$$

$$\frac{c_t}{c_{t-1}} = \frac{1+r_t-\delta_k}{1+\rho}$$

$$\Rightarrow R_t = r_t - \delta_k. \qquad R_0 - g = \rho(1+g)$$

Balanced growth -Spending

$$G_{-1} + \frac{G_0}{1+R_0} + \frac{G_1}{(1+R_0)(1+R_1)}$$

$$+ \frac{G_2}{(1+R_0)(1+R_1)(1+R_2)} + \ldots$$

$$= G_{-1} + \frac{G_0}{1+R_0}\left(1 + \frac{1+g}{1+R_0} + \left(\frac{1+g}{1+R_0}\right)^2 + \ldots\right)$$

$$= G_{-1} + \frac{G_0}{1+R_0}\left(\frac{1}{1-\frac{1+g}{1+R_0}}\right)$$

$$= G_{-1} + \frac{G_0}{R_0 - g}$$

$$= G_{-1} + \frac{G_0}{\rho(1+g)}.$$

Balanced growth– Money Creation

$$M_0 + \frac{M_1 - M_0}{1 + R_0} + \frac{M_2 - M_1}{(1 + R_0)(1 + R_1)}$$

$$+ \frac{M_3 - M_2}{(1 + R_0)(1 + R_1)(1 + R_2)} + \ldots$$

$$= M_0 + \frac{M_0(1 + g) - M_0}{1 + R_0} + \frac{M_0(1 + g)^2 - M_0(1 + g)}{(1 + R_0)(1 + R_0)}$$

$$+ \frac{M_0(1 + g)^3 - M_0(1 + g)^2}{(1 + R_0)(1 + R_0)(1 + R_0)} + \ldots$$

$$= \left(M_0 - \frac{M_0}{1 + R_0}\right)\left(1 + \frac{1 + g}{1 + R_0} + \left(\frac{1 + g}{1 + R_0}\right)^2 + \ldots\right)$$

$$= \frac{R_0 M_0}{R_0 - g} = M_0\left(\frac{\rho(1 + g) + g}{\rho(1 + g)}\right) = M_0\left(1 + \frac{g}{\rho(1 + g)}\right).$$

Wealth Constraint with balanced growth

$$M_0 + B_0 + \frac{G_0}{\rho(1+g)} = \frac{\tau y_0}{\rho(1+g)} + M_0\left(1 + \frac{g}{\rho(1+g)}\right)$$

$$\frac{G_0}{\rho(1+g)} = \frac{\tau y_0}{\rho(1+g)} + \frac{gM_0}{\rho(1+g)} - B_0.$$

$$B_0 = \frac{\tau y_0 + gM_0 - G_0}{\rho(1+g)}.$$

What happens if money is created at a faster rate than *g* in an attempt to lower taxes?

With Inflation

$$1+\pi_t \equiv \frac{P_{t+1}}{P_t}.$$

$$1+R_t = (1+\pi_t)(1+r_t-\delta_k).$$

$$M_{t+1} = M_t(1+\sigma),$$

if $\sigma = g$, then $\pi_t = 0$,

$$\frac{R_0 M_0}{R_0-\sigma} = M_0 + \frac{M_0(1+\sigma)-M_0}{1+R_0} + \frac{M_0(1+\sigma)^2-M_0(1+\sigma)}{(1+R_0)(1+R_0)} + \ldots.$$

Again, with no Inflation

$$\frac{R_0 M_0}{R_0-g} = M_0\left(\frac{\rho(1+g)+g}{\rho(1+g)}\right) = M_0\left(1+\frac{g}{\rho(1+g)}\right).$$

Revenue from Inflation

- When money supply growth rate σ rises,
  - can get more direct revenue from inflation tax,
- Second, higher inflation can lower real value of nominal debt,
  - by reducing real value of debt interest payments.
  - When inflation raises nominal interest rate above expected rate,
- Third: inflation can reduce stream of direct tax revenues,
  - by causing a lower endogenous growth rate.

Homework for May 3

- Read Chapter 20. Monetary Theory and Policy

  First 5 sections (Pages 842-871) Finish Chapter 20 for May 10 class (Mother’s day, will there be class?)

- Quiz 24