Introduction to Chapter 8 Dynamic Analysis and *AS*-*AD* (pp 322-371)

March 8, 2017

Homework for March 8 (and March 13)

- Read Chapter 8— Sections 8.4 to Section 8.6 for March 8 and through the end of the chaper for March 13.
- You will be required to set up and solve the recursive problem (Section 8.2) and apply it to the the centralized and decentralized models of sections 8.3 and 8.4.
- Also for March 13, you will replicate Figures 8.4, 8.7, and 8.8. (For the exam you will have to understand how these three figures change when there is a change in productivity or the time endowment.)

Quiz 14—the Recursive Model

1. Set up and solve the centralized model of Chapter 8.
   a. Write down the Bellman equation,
   b. the four constraints and
   c. the modified Bellman equation with constraints substituted in for consumption and leisure.
2. Write down the first order conditions and the envelope condition. Substitute in the envelope condition to get the two FOCs that represent the inter and intra temporal margins.

The Bellman Equation: A Recusive Policy Function

$$V(k_t) = \underset{c_t, x_t, k_{t+1}}{Max} : u(c_t, x_t) + \beta V(k_{t+1}).$$

$$V(k_t) = u(c_t, x_t) + \beta V(k_{t+1}) = \sum_{s=t}^{\infty} \beta^{s-t} u(c_s, x_s);$$

Can you show that this result is correct?

Plus "transversality condition" $\lim_{t \to \infty} [\beta^t V(k_t)] = 0.$

Set Up the Recursive Model (Centralized)

$$u(c_t, x_t) = \ln c_t + \alpha \ln x_t.$$

$$y_t = A l_t^{\gamma} k_t^{1-\gamma},$$

$$y_t = c_t + i_t.$$

$$k_{t+1} = i_t + (1 - \delta_k) k_t,$$

$$i_t = k_{t+1} - k_t (1 - \delta_k).$$

$$T = x_t + l_t.$$

$$V(k_t) = \underset{c_t, x_t, l_t, k_{t+1}}{Max} : u(c_t, x_t) + \beta V(k_{t+1}),$$

$$V(k_t) = \underset{l_t, k_{t+1}}{Max}: u\left(A l_t^{\gamma} k_t^{1-\gamma} - k_{t+1} + k_t (1 - \delta_k), T - l_t\right) + \beta V(k_{t+1}).$$

Some Questions

- How do you know that this is the centralized version?
- What variables will be left out of this solution?
- What are the first order conditions of the centralized model?
- What else do you need to compute MRS(c_t,c_{t+1}) and MRS (x_t,c_t) ?

First Order and Envelope Conditions

$$0 = \frac{\partial u(c_t, x_t)}{\partial c_t}\left(\gamma A l_t^{\gamma-1} k_t^{1-\gamma}\right) + \frac{\partial u(c_t, x_t)}{\partial x_t}(-1);$$

$$0 = \frac{\partial u(c_t, x_t)}{\partial c_t}(-1) + \beta\frac{\partial V(k_{t+1})}{\partial k_{t+1}}.$$

$$\frac{\partial V(k_t)}{\partial k_t} = \frac{\partial u(c_t, x_t)}{\partial c_t}\left[(1-\gamma)A l_t^{\gamma} k_t^{-\gamma} + (1-\delta_k)\right].$$

After substitution from the envelope condition you get two equilibrium conditions

$$0 = \frac{\partial u(c_t, x_t)}{\partial c_t}\left(\gamma A l_t^{\gamma-1} k_t^{1-\gamma}\right) + \frac{\partial u(c_t, x_t)}{\partial x_t}(-1);$$

$$\frac{\partial u(c_t, x_t)}{\partial c_t} = \beta \frac{\partial u(c_{t+1}, x_{t+1})}{\partial c_{t+1}}[(1-\gamma) A l_{t+1}^{\gamma} k_{t+1}^{-\gamma} + (1-\delta_k)].$$

Show that consumption growth is related to the ratio the return to capital and the rate of time preference.

Two Standard Margins: Intertemporal and Intratemporal

$$MRS_{c_t,c_{t+1}} = \frac{\frac{\partial u(c_t,x_t)}{\partial c_t}}{\beta\frac{\partial u(c_{t+1},x_{t+1})}{\partial c_{t+1}}} = 1 + (1-\gamma)Al_{t+1}^{\gamma}k_{t+1}^{-\gamma} - \delta_k$$

$$= 1 + MP_{k_{t+1}} - \delta_k.$$

$$\frac{c_{t+1}}{c_t} = \frac{1 + [(1-\gamma)A_G l_{t+1}^{\gamma}k_{t+1}^{-\gamma}] - \delta_k}{1+\rho}.$$

$$MP_{l_t} = \gamma A_G l_t^{\gamma-1}k_t^{1-\gamma} = \frac{\frac{\partial u(c_t,x_t)}{\partial x_t}}{\frac{\partial u(c_t,x_t)}{\partial c_t}} = \frac{\frac{\alpha}{x_t}}{\frac{1}{c_t}} = MRS_{x,c}.$$

The decentralized model—the consumer
Note that in budget constraint (Equation 8.26) he still includes profit which we know is zero—here it would drop out of FOC anyway.

$$s_t = k^s_{t+1} - k^s_t(1-\delta_k).$$

$$c^d_t = w_t l^s_t + r_t k^s_t - k^s_{t+1} + k^s_t(1-\delta_k).$$

$$V(k^s_t) = \underset{c^d_t, x_t, l^s_t, k^s_{t+1}}{Max} : u\left(c^d_t, x_t\right) + \beta V(k^s_{t+1}),$$

$$V(k^s_t) = \underset{l^s_t, k^s_{t+1}}{Max}: u[w_t l^s_t + r_t k^s_t - k^s_{t+1} + k^s_t(1-\delta_k), T - l_t] + \beta V(k^s_{t+1}).$$

Consumer equilibrium

$$0 = \frac{\partial u\left(c_t^d, x_t\right)}{\partial c_t} w_t - \frac{\partial u\left(c_t^d, x_t\right)}{\partial x_t};$$

$$0 = \frac{-\partial u\left(c_t^d, x_t\right)}{\partial c_t^d} + \beta \frac{\partial V(k_{t+1}^s)}{\partial k_{t+1}^s}.$$

$$Envelope \ : \ \frac{\partial V(k_t^s)}{\partial k_t^s} = \frac{\partial u\left(c_t^d, x_t\right)}{\partial c_t^d}(1 + r_t - \delta_k).$$

$$\Rightarrow \frac{c_{t+1}^d}{c_t^d} = \frac{1 + r_{t+1} - \delta_k}{1 + \rho}, \ w_t = \frac{\frac{\alpha}{x_t}}{\frac{1}{c_t^d}}.$$

The decentralized model—the firm

$$TECHNOLOGY\ :\ y_t = A_G \left(l_t^d\right)^{\gamma} \left(k_t^d\right)^{1-\gamma},$$

$$\underset{y_t, l_t^d, k_t^d}{Max} \quad \Pi_t = y_t - w_t l_t^d - r_t k_t^d;$$

$$\underset{l_t^d, k_t^d}{Max} \quad \Pi_t = A_G \left(l_t^d\right)^{\gamma} \left(k_t^d\right)^{1-\gamma} - w_t l_t^d - r_t k_t^d\ :$$

$$w_t = \gamma A_G \left(l_t^d\right)^{\gamma-1} \left(k_t^d\right)^{1-\gamma},$$

$$r_t = (1-\gamma) A_G \left(l_t^d\right)^{\gamma} \left(k_t^d\right)^{-\gamma}.$$

Can you show that profits = 0?

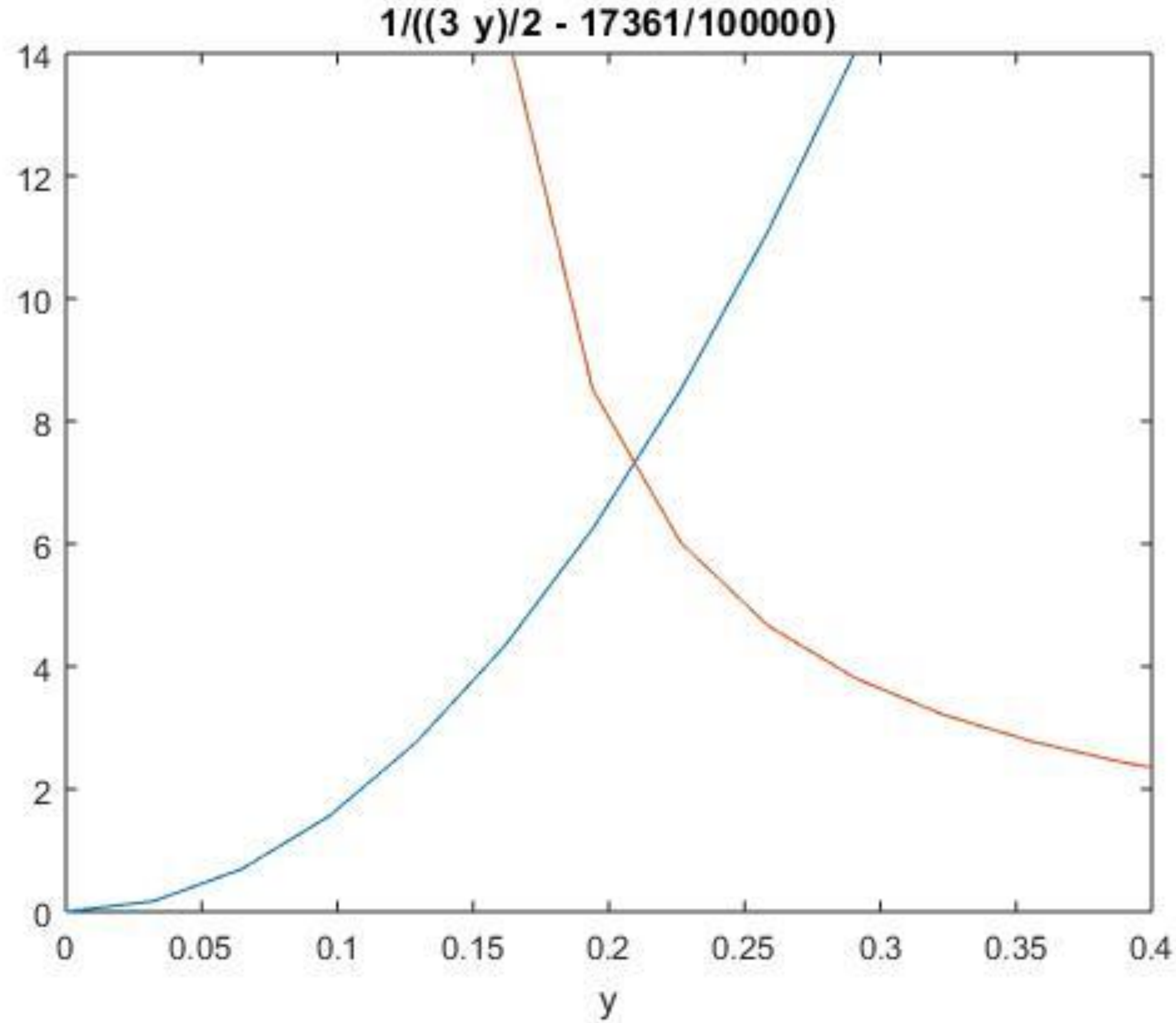
Hint: $w_t l_t^d = \gamma y_t,$ and $r_t k_t^d = (1-\gamma) y_t$

Matlab Program to Replicate Figure 8.4

```
% Model_Gillman_ch8.m
% This model includes log utility and cobb douglas production
% This program was last updated March 7, 2017
clc
clear
%***************************************
% Calibrated Parameters
%***************************************
parameter_names = ['gamma',' ','delta',' ', 'rho',' ', 'Ag',' ', 'alpha',' ', 'time' ];
gam = 1/3 ;      % labor's share of output
delt = 0.03;     % steady state capital depreciation rate
rho = 0.03;     % rate of time preference
ag = 0.15;        % Technology Factor, also called productivity factor
alph = 0.5;      % Weight on leisure in log utility
tim = 1;      % Time available for work and leisure
```

```
parameter_values = [gam delt rho ag alph tim];
rk = rho + delt ;
k = 2.3148 ;
%*****************************
% output
%*****************************
 %********** AD-AS ***Figure 8.4**************
syms y
f(y) = (y^((1-gam)/gam)/((gam*ag^(1/gam))*k^((1-gam)/gam))); % AS eq 8.62
g(y) = tim/(y*(1+alph)-k*(rho+(1+alph)*delt)) ;                % AD eq 8.57
LIMS = [0 .4 0 14];
ezplot(f,LIMS)
hold
ezplot(g,LIMS)
```

Figure 8.4



Chapter 8 takes the solution of *k* as given.
Chapter 10 shows ways to compute *k*

$$k_t = \frac{T\gamma A_G^{\frac{1}{\gamma}}\left[\frac{(1-\gamma)}{\rho+\delta_k}\right]^{\frac{1-\gamma}{\gamma}}}{(\gamma+\alpha)\left(\frac{\rho+\delta_k}{(1-\gamma)}\right)-\alpha\delta_k}$$

All Other Variables are a Function of k and parameters

$$r_t = \rho + \delta_k;$$

$$l_t = \left[\frac{\rho + \delta_k}{(1-\gamma)A_G} \right]^{\frac{1}{\gamma}} k_t;$$

$$w_t = \gamma A_G \left[\frac{(1-\gamma)A_G}{\rho + \delta_k} \right]^{\frac{1-\gamma}{\gamma}};$$

$$y_t = A_G \left[\frac{\rho + \delta_k}{(1-\gamma)A_G} \right] k_t;$$

$$i_t = \delta_k k_t;$$

$$c_t = A_G \left[\frac{\rho + \delta_k}{(1-\gamma)A_G} \right] k_t - \delta_k k_t.$$

$$x_t = T - l = T - \left[\frac{\rho + \delta_k}{(1-\gamma)A_G} \right]^{\frac{1}{\gamma}} k_t;$$

Homework for March 13

- Read through the end of Chapter 8
- Replicate Figures 8.4, 8.7, and 8.8. (For the exam you will have to understand how these three figures change when there is a change in productivity or the time endowment.)
- Continue to turn in old quizzes. Feel free to cooperate in homework, but be sure that you have the ability to answer questions on you own by March 29.