Chapters 2 - 7: Review Elementary Microfoundation Modeling (pp 64 to 313)

Introduction to Chapter 8 (pp 322-371)

March 6, 2017

Homework for March 6

- Read Chapter 8— Sections 8.1 to Section 8.4 for March 6 and through the end of the chaper for March 8.
- You will be required to set up and solve the recursive problem (Section 8.2) and apply it to both the centralized and decentralized models in Sections 8.3 and 8.4

Review: Setting Up and Solving the Model

- Today, we are going to review what we learned about how to solve the models of Chapters 2-7.

Quiz 13

- Set up and solve the baseline centralized model of chapter 2. Solve using general parameter values for *T*, *AG*, α and γ. Fix the capital stock at $k = 1$.
  - Set parameter values T=1, A_G=1, α=1/2 and γ=1/3. Compute *c, u, l,* and *x.*
  - What happens to *c, u, l,* and *x* when we double A_G in the baseline model?
- Set up and solve the baseline model (decentralized) of chapter 2. Solve using parameter values for T=24, A_G=1, α=1 and γ=1/2.
  - Calculate the equilibrium wage, *w*.
  - What is the price of consumption?
  - Write equations for labor supply, labor demand, consumption supply and consumption demand.

Quiz 13 Continued

- Set up and solve the baseline centralized model of chapter 5. Solve using general parameter values for T, AG, α and γ. Fix the capital stock at $k = 1$.
  - What happens to c, u, l, and x when we double AG in the baseline model?

- Set up and solve the model of the small open endowment economy of Chapter 7. Why is the relationship between the rate of time preference and the interest rate important?

Chapter 8 Combines chapters 2 and 5

- We do not consider growth until chapter 9 and we do not solve directly for the capital stock until chapter 10.
- Here we learn how to convert our dynamic 2 period problem into one in which the agents are 'infinitely-lived.'
- A recursive model is a special case of an equation system where the endogenous variables are determined one at a time in sequence.
- In our case, the solution requires an assumption about the initial value for capital.

Some Background

- What was the Rational Expectations" revolution?
- Why doesn't the author use the term "Rational Expectations?“
- What are the tradeoffs that the household faces?
- What are the household's decision variables?
- What is a policy function in the context of this model?

The Bellman Equation: A Recusive Policy Function

$$V(k_t) = \underset{c_t, x_t, k_{t+1}}{Max} : u(c_t, x_t) + \beta V(k_{t+1}).$$

$$V(k_t) = u(c_t, x_t) + \beta V(k_{t+1}) = \sum_{s=t}^{\infty} \beta^{s-t} u(c_s, x_s);$$

Can you show that this result is correct?

Plus "transversality condition" $\lim_{t \to \infty} [\beta^t V(k_t)] = 0.$

Set Up the Recursive Model (Centralized)

$$u(c_t, x_t) = \ln c_t + \alpha \ln x_t.$$

$$y_t = A l_t^{\gamma} k_t^{1-\gamma},$$

$$y_t = c_t + i_t.$$

$$k_{t+1} = i_t + (1 - \delta_k) k_t,$$

$$i_t = k_{t+1} - k_t (1 - \delta_k).$$

$$T = x_t + l_t.$$

$$V(k_t) = \underset{c_t, x_t, l_t, k_{t+1}}{Max} : u(c_t, x_t) + \beta V(k_{t+1}),$$

$$V(k_t) = \underset{l_t, k_{t+1}}{Max} : u\left(A l_t^{\gamma} k_t^{1-\gamma} - k_{t+1} + k_t (1 - \delta_k), T - l_t\right) + \beta V(k_{t+1}).$$

Some Questions

- How do you know that this is the centralized version?
- What variables will be left out of this solution?
- What are the first order conditions of the centralized model?
- What else do you need to compute MRS(c_t,c_{t+1}) and MRS (x_t,c_t) ?

First Order and Envelope Conditions

$$0 = \frac{\partial u(c_t, x_t)}{\partial c_t}\left(\gamma A l_t^{\gamma-1} k_t^{1-\gamma}\right) + \frac{\partial u(c_t, x_t)}{\partial x_t}(-1);$$

$$0 = \frac{\partial u(c_t, x_t)}{\partial c_t}(-1) + \beta \frac{\partial V(k_{t+1})}{\partial k_{t+1}}.$$

$$\frac{\partial V(k_t)}{\partial k_t} = \frac{\partial u(c_t, x_t)}{\partial c_t}[(1-\gamma) A l_t^{\gamma} k_t^{-\gamma} + (1-\delta_k)].$$

After substitution from the envelope condition you get two equilibrium conditions

$$0 = \frac{\partial u(c_t, x_t)}{\partial c_t}\left(\gamma A l_t^{\gamma-1} k_t^{1-\gamma}\right) + \frac{\partial u(c_t, x_t)}{\partial x_t}(-1);$$

$$\frac{\partial u(c_t, x_t)}{\partial c_t} = \beta \frac{\partial u(c_{t+1}, x_{t+1})}{\partial c_{t+1}}[(1-\gamma) A l_{t+1}^{\gamma} k_{t+1}^{-\gamma} + (1-\delta_k)].$$

Show that consumption growth is related to the ratio the return to capital and the rate of time preference.

Two Standard Margins: Intertemporal and Intratemporal

$$MRS_{c_t,c_{t+1}} = \frac{\frac{\partial u(c_t,x_t)}{\partial c_t}}{\beta \frac{\partial u(c_{t+1},x_{t+1})}{\partial c_{t+1}}} = 1 + (1-\gamma)Al_{t+1}^{\gamma}k_{t+1}^{-\gamma} - \delta_k$$

$$= 1 + MP_{k_{t+1}} - \delta_k.$$

$$\frac{c_{t+1}}{c_t} = \frac{1 + [(1-\gamma)A_G l_{t+1}^{\gamma} k_{t+1}^{-\gamma}] - \delta_k}{1+\rho}.$$

$$MP_{l_t} = \gamma A_G l_t^{\gamma-1} k_t^{1-\gamma} = \frac{\frac{\partial u(c_t,x_t)}{\partial x_t}}{\frac{\partial u(c_t,x_t)}{\partial c_t}} = \frac{\frac{\alpha}{x_t}}{\frac{1}{c_t}} = MRS_{x,c}.$$

Homework for March 8 (and March 13)

- Read Chapter 8— Sections 8.4 to Section 8.6 for March 8 and through the end of the chaper for March 13.
- You will be required to set up and solve the recursive problem (Section 8.2) and apply it to the the centralized and decentralized models of sections 8.3 and 8.4.
- Also for March 13, you will replicate Figures 8.4, 8.7, and 8.8. (For the exam you will have to understand how these three figures change when there is a change in productivity or the time endowment.)