Chapters 9 and 10: GE model continued
(pp 372-439)

March 22, 2017

Homework for March 22

- Finish Reading Chapter 9
- Complete Quiz 17.
  - Set up the GE recursive model with a labor tax.
  - Write down the utility function, the four constraints and the modified Bellman equation for the decentralized solution.
  - Take first order conditions and derive the equations for aggregate demand and aggregate supply.
- Read Chapter 10.

Quiz 17

1. Set up and solve the GE recursive model with a labor tax. Write down the Bellman equation, the utility function, the four constraints.
2. Write down the modified Bellman equation for the decentralized solution.
3. Take first order conditions, the envelope condition and derive the equations for aggregate demand and aggregate supply.

Quiz 17, #1 Bellman equation and constraints

$$V(k_t) = \underset{c_t, x_t, k_{t+1}}{Max} : u(c_t, x_t) + \beta V(k_{t+1}).$$

$$u(c_t, x_t) = \ln c_t + \alpha \ln x_t.$$

$$y_t = A l_t^{\gamma} k_t^{1-\gamma},$$

$$y_t = c_t + i_t.$$

$$i_t = k_{t+1} - k_t(1 - \delta_k).$$

$$T = x_t + l_t.$$

Quiz 17, #2 Budget Constraint with Labor Tax and Bellman Equation for Decentralized Model

$$c_t^d = w_t(1-\tau_l)l_t^s + r_t k_t + G_t - k_{t+1} + k_t(1-\delta_k).$$

$$V(k_t^s) =$$

$$\underset{l_t^s, k_{t+1}^s}{Max}: \; u[(1-\tau_l)w_t l_t^s + r_t k_t^s + G_t - k_{t+1}^s + k_t^s(1-\delta_k), T - l_t] + \beta V(k_{t+1}^s).$$

Quiz 17, #3 FOC, Envelope Condition

$$0 = \frac{\partial u\left(c_t^d, x_t\right)}{\partial c_t} w_t(1-\tau_l) - \frac{\partial u\left(c_t^d, x_t\right)}{\partial x_t};$$

$$0 = \frac{-\partial u\left(c_t^d, x_t\right)}{\partial c_t^d} + \beta \frac{\partial V(k_{t+1}^s)}{\partial k_{t+1}^s}.$$

$$Envelope \; : \; \frac{\partial V(k_t^s)}{\partial k_t^s} = \frac{\partial u\left(c_t^d, x_t\right)}{\partial c_t^d}(1 + r_t - \delta_k).$$

$$\Rightarrow \frac{c_{t+1}^d}{c_t^d} = \frac{1 + r_{t+1} - \delta_k}{1+\rho}, \; w_t = \frac{\frac{\alpha}{x_t}}{\frac{1}{c_t^d(1-\tau_l)}}.$$

Aggregate Demand = $c^d + i^d$

$$AD\ :\ y_t^d = c_t^d + i_t = \left(\frac{1}{1+\frac{\alpha}{1-\tau_l}}\left[w_t T + \rho k_t\right]\right) + \delta_k k_t,$$

$$y_t^d = \frac{w_t T + k_t\left[\rho + \left(1+\frac{\alpha}{1-\tau_l}\right)\delta_k\right]}{1+\frac{\alpha}{1-\tau_l}}.$$

$$Inversely\ :\ \frac{1}{w_t} = \frac{T}{y_t^d\left(1+\frac{\alpha}{1-\tau_l}\right) - k_t\left[\rho + \left(1+\frac{\alpha}{1-\tau_l}\right)\delta_k\right]}.$$

Aggregate Supply (same as no labor income tax)

$$l_t^d = \left(\frac{\gamma A_G}{w_t}\right)^{\frac{1}{1-\gamma}} k_t.$$

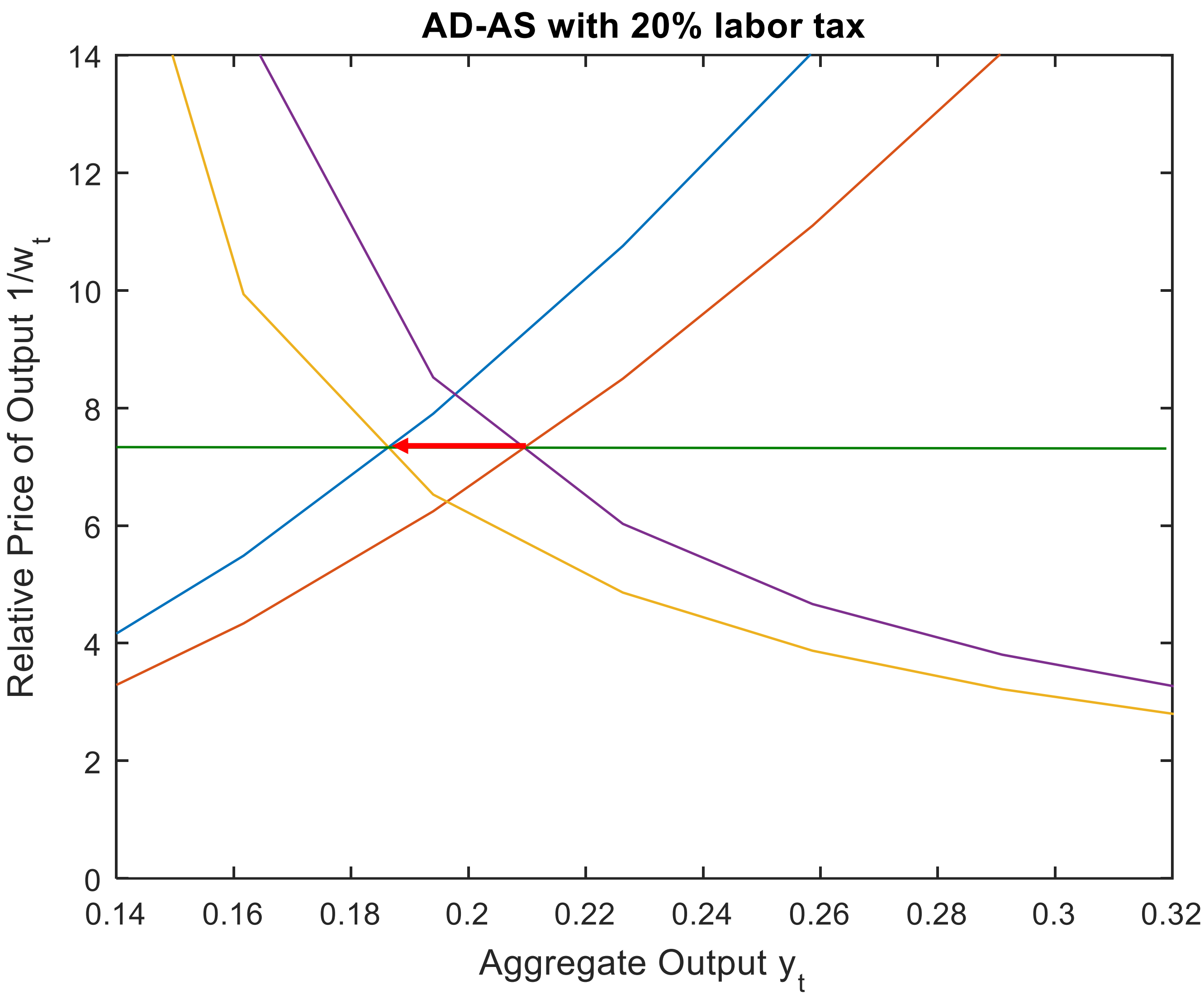
$$y_t^s = A_G \left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma}.$$

$$AS: y_t^s = A_G (k_t)^{1-\gamma} \left(\frac{\gamma A_G}{w_t}\right)^{\frac{\gamma}{1-\gamma}} (k_t)^{\gamma} = A_G^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t.$$

$$Inversely: \frac{1}{w_t} = \frac{(y_t^s)^{\frac{1-\gamma}{\gamma}}}{\gamma A_G^{\frac{1}{\gamma}} (k_t)^{\frac{1-\gamma}{\gamma}}}.$$

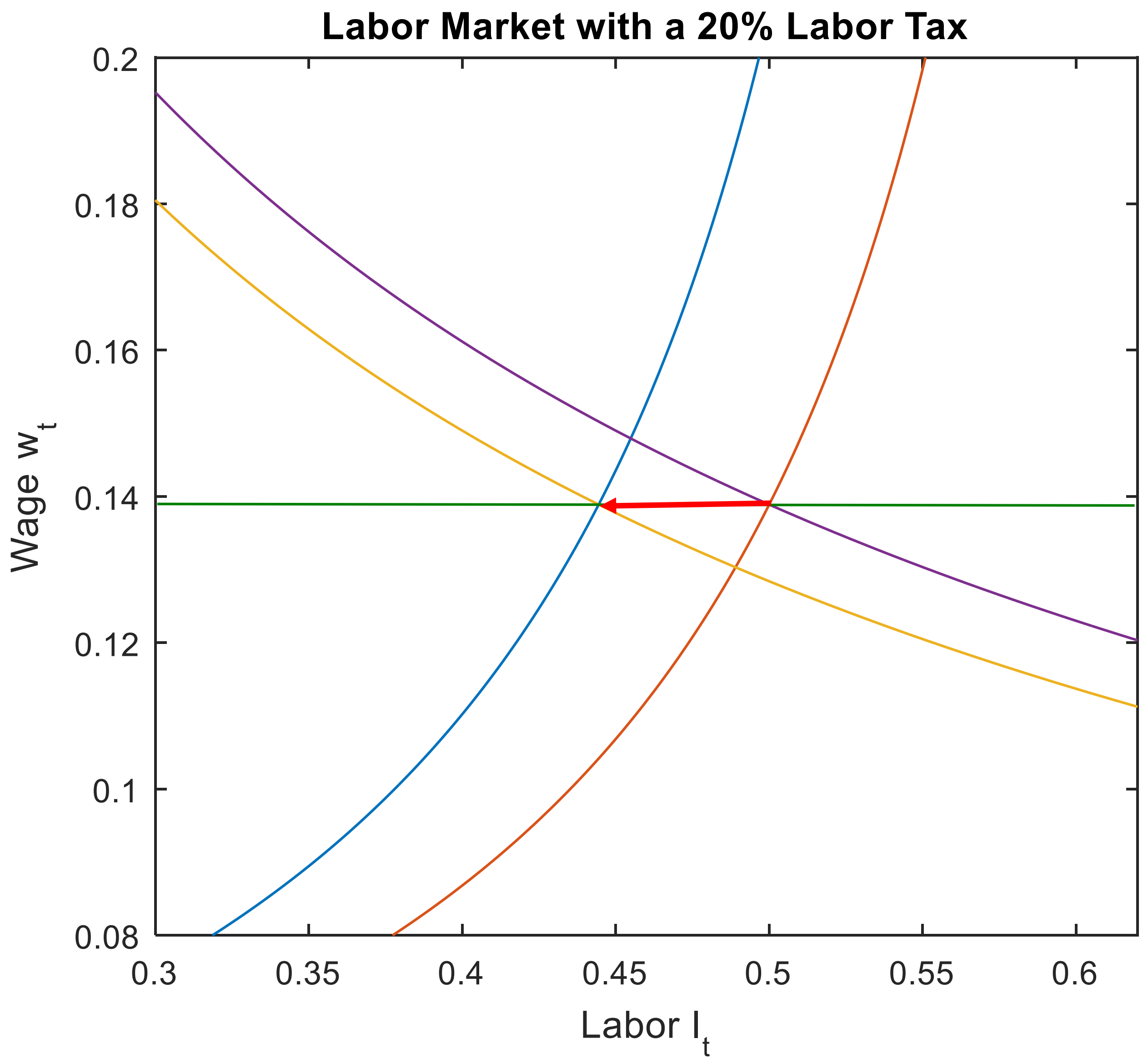
How does AD-AS change with a labor tax? Figure 9.12

How does AD-AS change with a labor tax? Figure 9.12



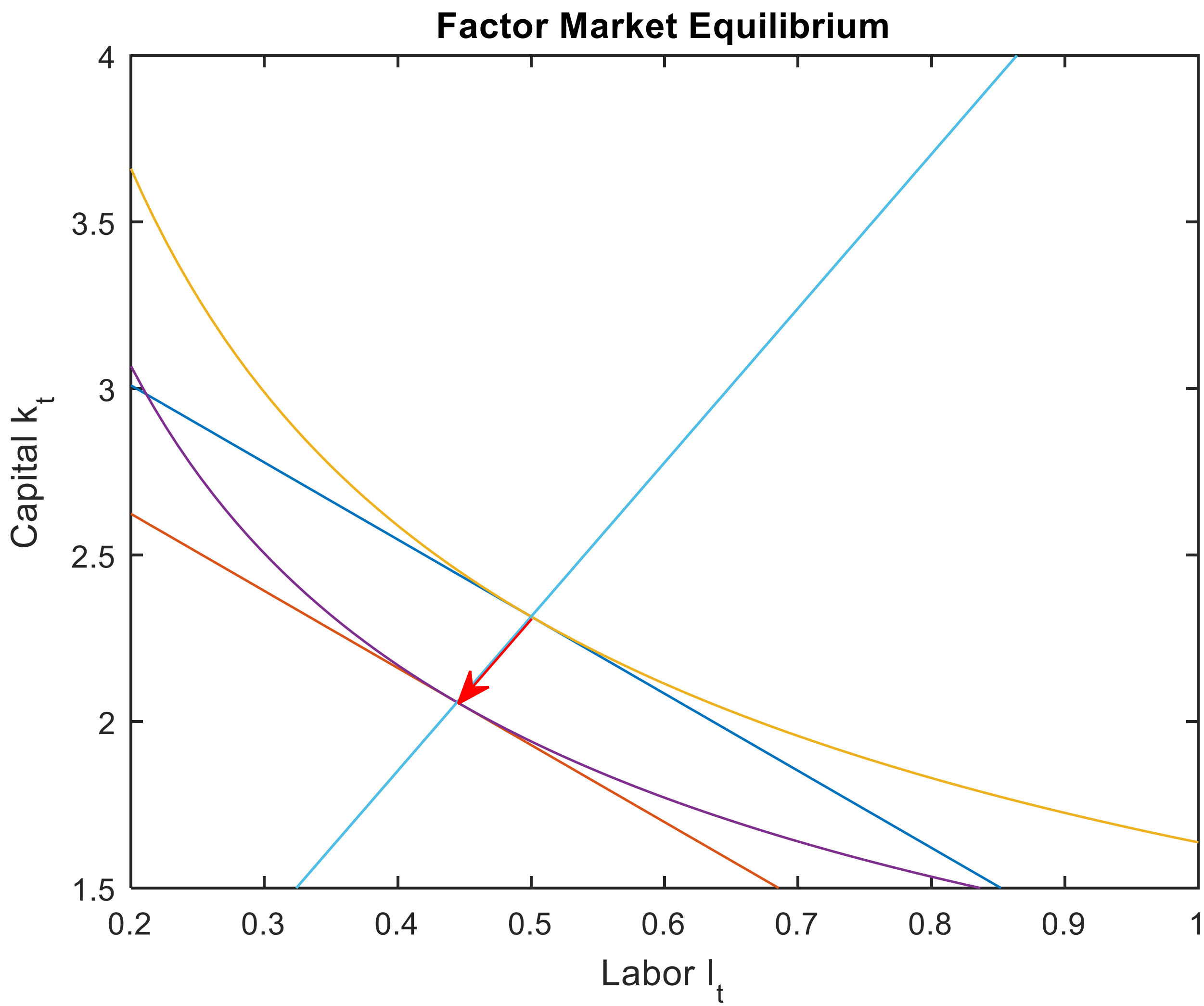
How does the labor market change with a labor tax? Figure 9.14

How does the labor market change with a labor tax? Figure 9.14



How does the factor market equilibrium change with a labor tax? Figure 9.15

How does the factor market equilibrium change with a labor tax? Figure 9.15



How does the general equilibrium change with a labor tax? Figure 9.16

How does the general equilibrium change with a labor tax? Figure 9.16

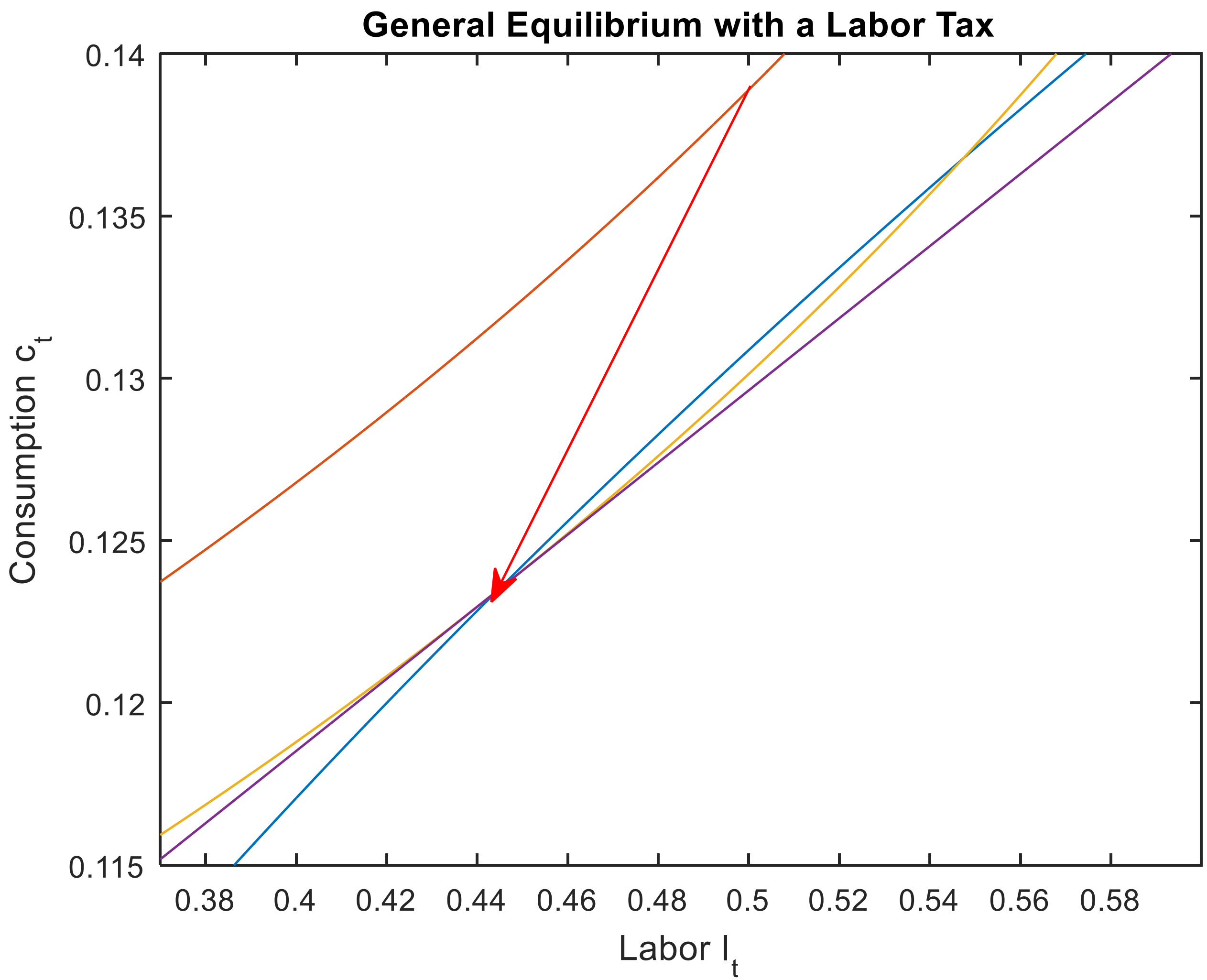
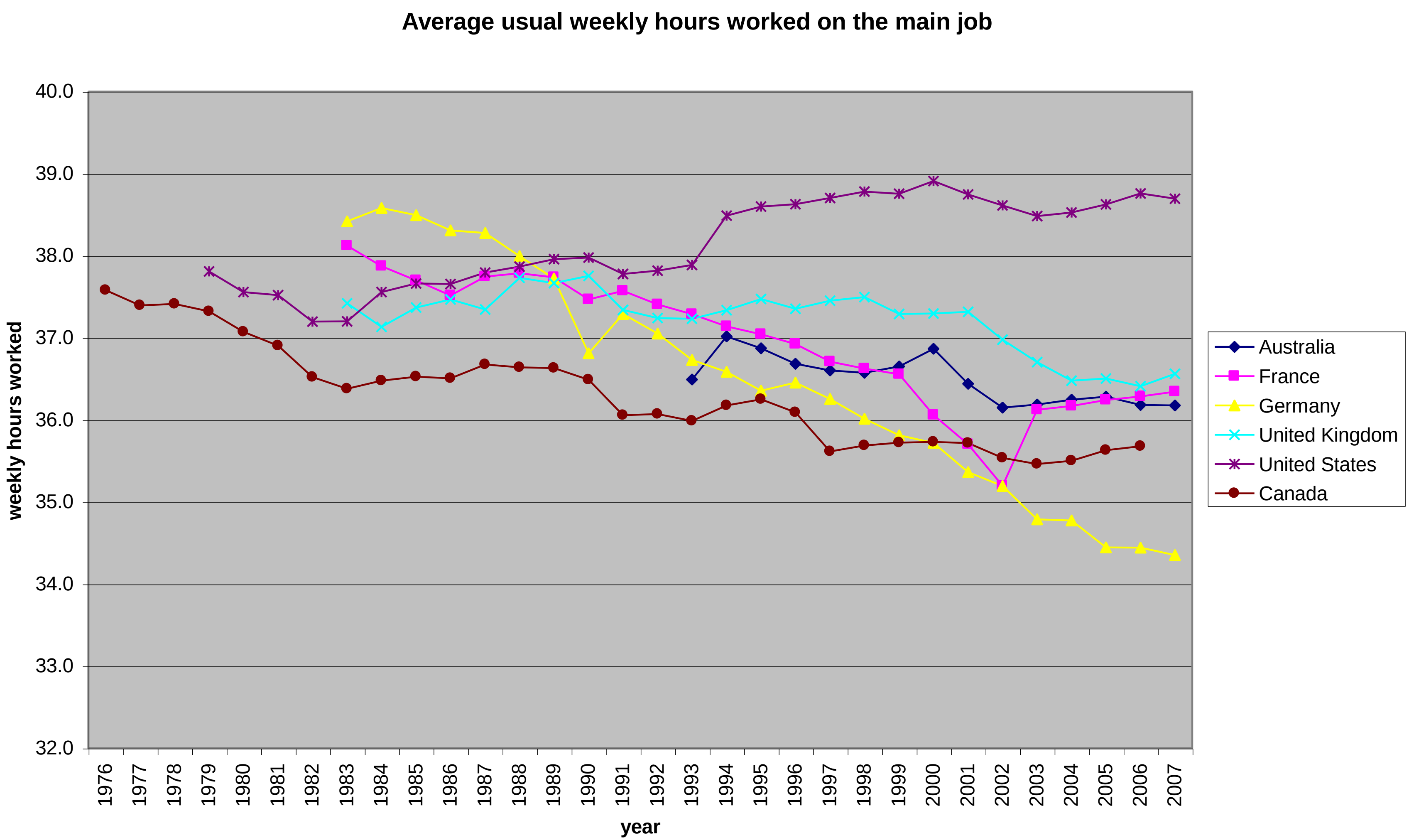


Figure 9.17



Average usual weekly hours worked on the main job
weekly hours worked
40.0
39.0
38.0
37.0
36.0
35.0
34.0
33.0
32.0
1976
1977
1978
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1980
1981
1982
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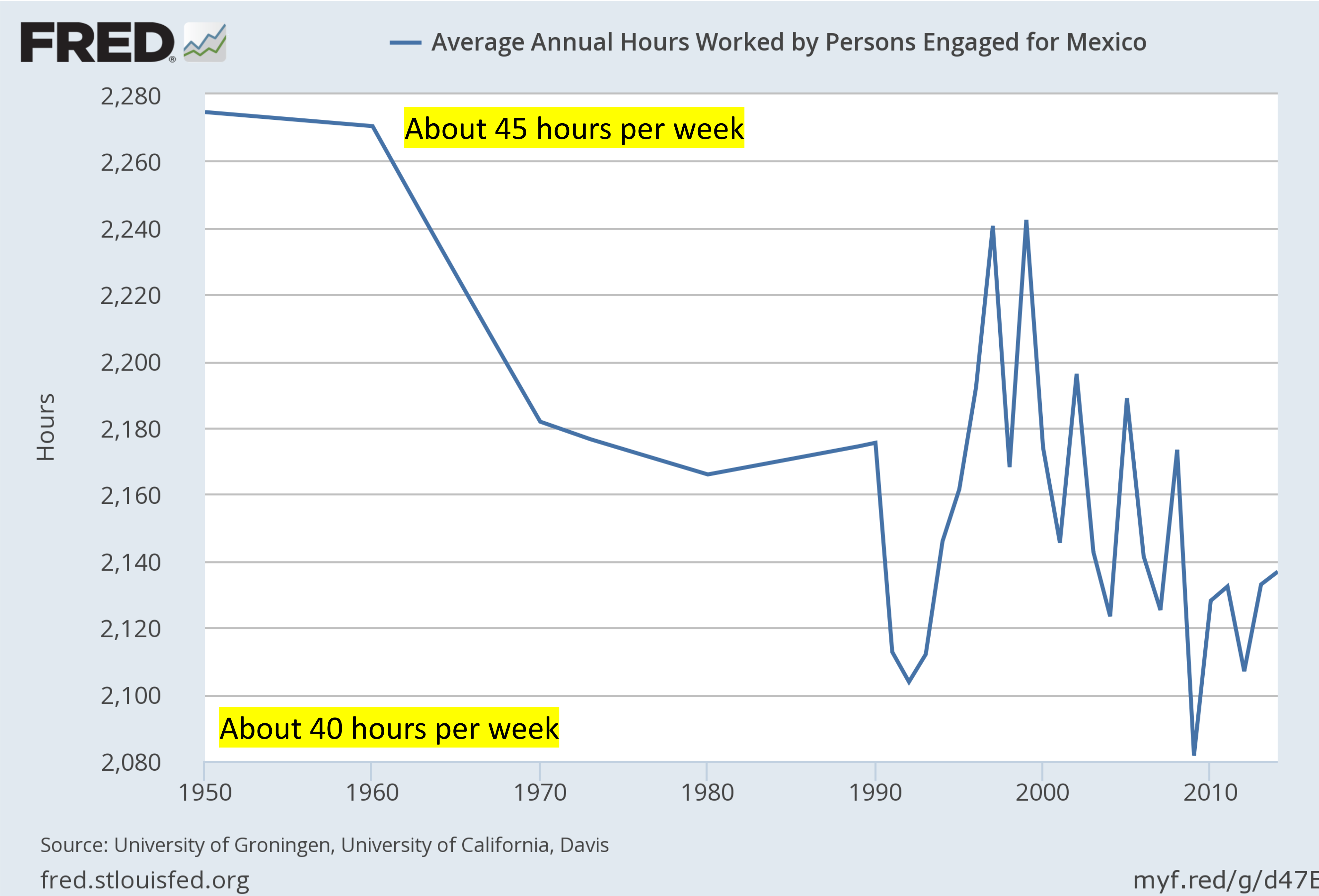
2007
year
Australia
France
Germany
United Kingdom
United States
Canada


FRED
Average Annual Hours Worked by Persons Engaged for Mexico
About 45 hours per week
About 40 hours per week
Hours
2,280
2,260
2,240
2,220
2,200
2,180
2,160
2,140
2,120
2,100
2,080
1950
1960
1970
1980
1990
2000
2010
Source: University of Groningen, University of California, Davis
fred.stlouisfed.org
myf.red/g/d47E

Chapter 10: Computing the Capital Stock as a Function of Parameters (no taxes here)

Starting with the equations for AD and AS

$$y_t^d = \frac{w_t T + k_t\left[\rho + \left(1 + \frac{\alpha}{1-\tau_l}\right)\delta_k\right]}{1 + \frac{\alpha}{1-\tau_l}}$$

$$y_t^s = A_G^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t$$

Two equations in two unknowns, *k* and *w*

Set AD=AS and find a solution for *w*

Use the marginal conditions from profit maximization

$$1 = \frac{c_{t+1}}{c_t} = \frac{1 + r_t - \delta_k}{1 + \rho},$$

$$\Rightarrow r_t = \rho + \delta_k.$$

$$\rho + \delta_k = r_t = (1 - \gamma) A_G \left(\frac{l_t}{k_t} \right)^{\gamma},$$

$$\Rightarrow \frac{l_t}{k_t} = \left[\frac{\rho + \delta_k}{(1 - \gamma) A_G} \right]^{\frac{1}{\gamma}}.$$

$$w_t = \gamma A_G \left(\frac{l_t}{k_t} \right)^{\gamma - 1},$$

$$\Rightarrow w_t = \gamma A_G \left[\frac{\rho + \delta_k}{(1 - \gamma) A_G} \right]^{\frac{\gamma - 1}{\gamma}}.$$

$$w_t = \gamma A_G \left[\frac{\rho + \delta_k}{(1-\gamma)A_G} \right]^{\frac{\gamma-1}{\gamma}}.$$

$$y_t^d - y_t^s = \frac{w_t T + k_t[\rho + (1+\alpha)\delta_k]}{1+\alpha} - A_G^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t = 0$$

Substitute for w_t, and you get:

$$0 = \frac{\gamma A_G \left[\frac{\rho+\delta_k}{(1-\gamma)A_G} \right]^{\frac{\gamma-1}{\gamma}} T + k_t[\rho + (1+\alpha)\delta_k]}{1+\alpha}$$

$$- A_G^{\frac{1}{1-\gamma}} \left(\frac{(\gamma)^{\frac{\gamma}{1-\gamma}}}{(\gamma A_G)^{\frac{\gamma}{1-\gamma}} \left[\frac{\rho+\delta_k}{(1-\gamma)A_G} \right]^{\frac{-\gamma}{\gamma}}} \right) k_t.$$

And with a little bit of arithmetic, you get

$$k_t = \frac{T\gamma A_G^{\frac{1}{\gamma}} \left[\frac{(1-\gamma)}{\rho+\delta_k} \right]^{\frac{1-\gamma}{\gamma}}}{(\gamma+\alpha)\left(\frac{\rho+\delta_k}{(1-\gamma)} \right) - \alpha\delta_k}$$

Alternative Solutions

- The first order conditions, the envelope condition and the constraints are used to set up a system with the same number of variables as equations.
- Substitution can be used in a large number of ways to solve for a particular variable.
- Here we start with capital and find that it is easy to compute all the other variables once we have determined capital.

All Other Variables are a Function of k and parameters

$$r_t = \rho + \delta_k;$$

$$l_t = \left[\frac{\rho + \delta_k}{(1-\gamma)A_G} \right]^{\frac{1}{\gamma}} k_t;$$

$$w_t = \gamma A_G \left[\frac{(1-\gamma)A_G}{\rho + \delta_k} \right]^{\frac{1-\gamma}{\gamma}};$$

$$y_t = A_G \left[\frac{\rho + \delta_k}{(1-\gamma)A_G} \right] k_t;$$

$$i_t = \delta_k k_t;$$

$$c_t = A_G \left[\frac{\rho + \delta_k}{(1-\gamma)A_G} \right] k_t - \delta_k k_t.$$

$$x_t = T - l = T - \left[\frac{\rho + \delta_k}{(1-\gamma)A_G} \right]^{\frac{1}{\gamma}} k_t;$$

Homework for March 27

- Finish reading Chapter 10
- Explain two ways to derive the formula for capital in chapter 10.
  - The book starts with AS-AD and then solves for capital.
  - It also derives the result starting with the two marginal conditions.
- Quiz 18 will include 3 questions from the first 17 quizzes. You will complete it in class.