# Chapter 5 (pp 208-243)

February 22, 2017

# Homework for Feb 22

- Finish reading Chapter 5
- Replicate figures 5.7, 5.8, and 5.9
- Prepare questions for Bank of Mexico sesión.

# Quiz 10 February 22, 2017

1. Set up the two period model with capital and fixed labor.
   a. What is the role of β in the utility function?
2. Calculate the first order conditions needed to compute the equilibrium solution to this model.
   a. Write the solution for the k1 in terms of the model parameters.
3. Set up the utility function to compute the decentralized solution.
   a. What is the combined budget constraint for the consumer?
   b. Express this constraint in a way that shows that the present value of income = present value of expenditures.
4. Derive the supply and demand curves for capital in this simple two period model with capital.
5. Explain why the relative price of capital is 1+r in this simple model?

# The agent's maximization problem

$$u(c_0, c_1) = \ln c_0 + \beta \ln c_1$$

$$y_0 = c_0 + k_1$$

$$c_1 = A_G (k_1)^{1-\gamma}$$

We are treating labor and leisure as fixed and the initial endowment of output is exogenous. Capital deprecation rate is 100%. It is completely used up in the production process.

# Question #2a

- What is the general expression for the capital stock, that is, before calibration?

$$k_1 = y_0 \beta(1-\gamma)/(1+ \beta(1-\gamma))$$

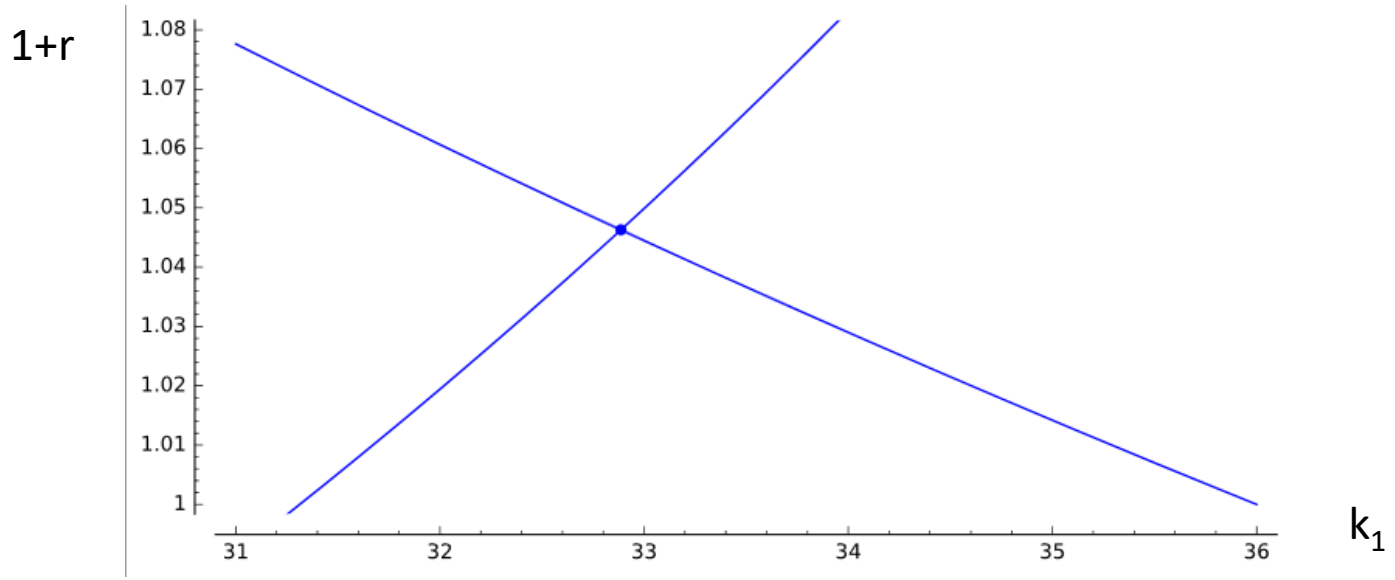
# Quiz 10 February 22, 2017

1. Set up the two period model with capital and fixed labor.
   a. What is the role of β in the utility function?
2. Calculate the first order conditions needed to compute the equilibrium solution to this model.
   a. Write the solution for the k1 in terms of the model parameters.
3. Set up the utility function to compute the decentralized solution.
   a. What is the combined budget constraint for the consumer?
   b. Express this constraint in a way that shows that the present value of income = present value of expenditures.
4. Derive the supply and demand curves for capital in this simple two period model with capital.
5. Explain why the relative price of capital is 1+r in this simple model?

# The Market for Capital

- What is the price of capital?
- Plot Figure 5.4 (Market for capital)
- Solve for $k^s$ in terms of (1+r) and Π
- Solve for $k^d$ in terms of (1+r)
- Use $k^d$ to calculate Π in terms of (1+r)
- Invert $k^s$ and $k^d$ and plot Figure 5.6

# Figure 5.4 The Market for Capital



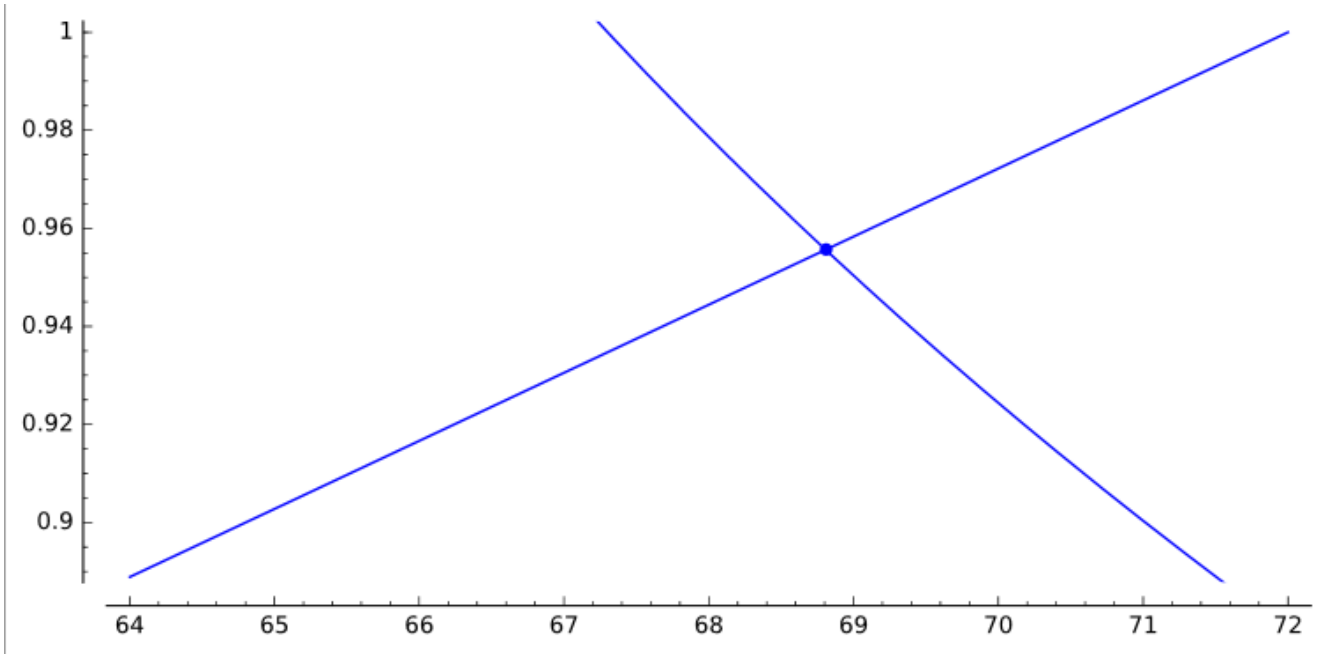

plot(((e^8.3532)/c0)^(1/.98), 0, 130, ymax=130)+plot(12*(100-c0)^.5, 0, 130)+point((67.114,68.816), size=30)

# The Market for Future Consumption

- What is the price of future consumption?
- Plot Figure 5.5 (market for $c_1$)
- Solve for $c_1^s$ in terms of (1+r)
- Solve for $c^d$ in terms of (1+r)
- Use budget constraint to generate $c^d$ function
- Solve in terms of 1/(1+r) and plot Figure 5.5

# Figure 5.5 The market for future consumption

1/(1+r)



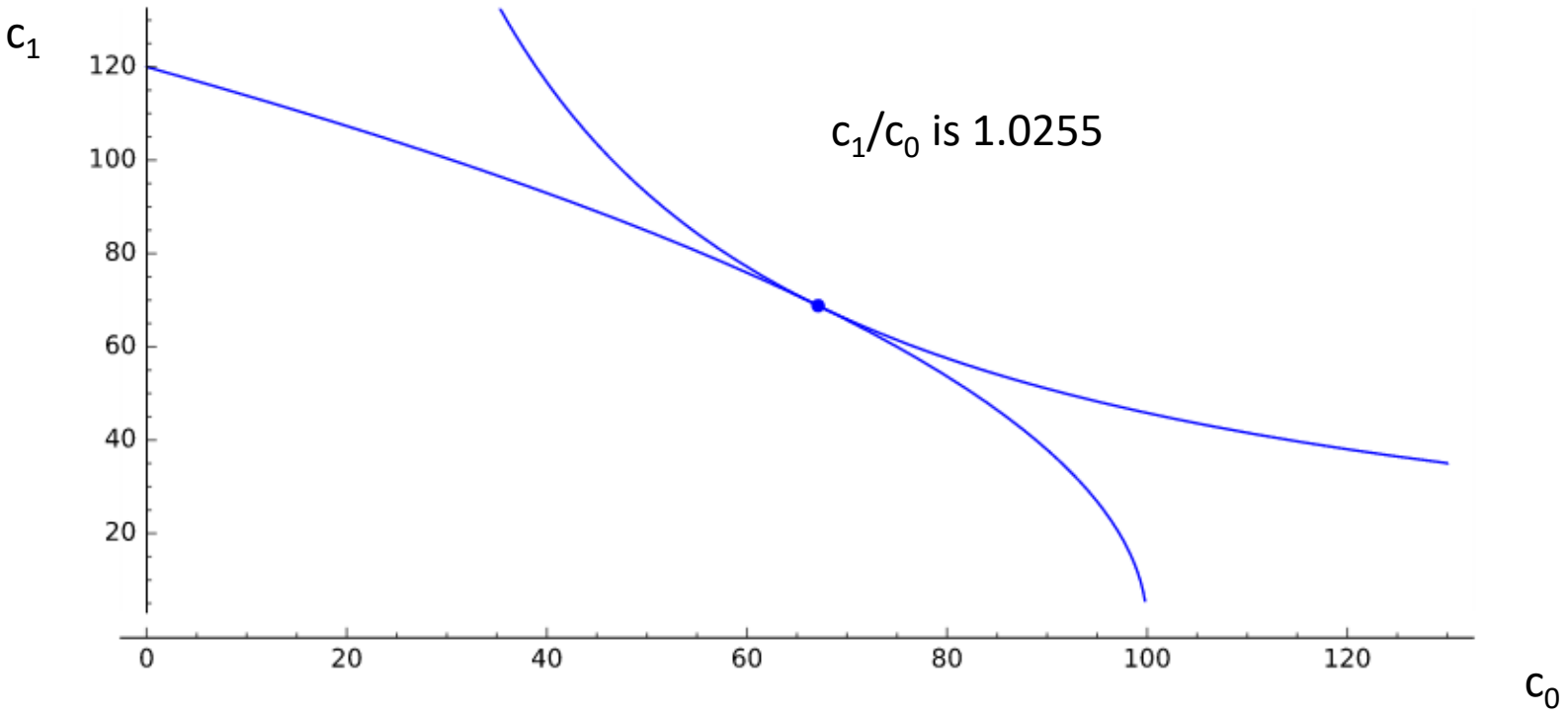

$c_1$

plot(c1/72, 64, 72, ymin=.89, ymax=1)+ plot(.02805*(c1-(c1^2-4*17.82*49.49)^.5), 64, 72, ymin=.89, ymax=1) + point((68.81,.9557), size=30)

# Graph the equilibrium in ($c_0$, $c_1$)

- Calibrate the model, assuming values for the productivity factor, the discount factor and the capital share in the Cobb-Douglas production function.
- Solve for the first order conditions in terms of the capital stock.
- Compute the expression for the capital stock and use it express utility in terms of $c_0$ and $c_1$.
- Use it to express the production function for $c_1$ in terms of $c_0$. Using the baseline assumptions in Chapter 5, we get
  - c1 = e^8.3532)/c0)^(1/.98)
  - c1 = 12*(100-c0)^.5
  - Equilibrium (c0,c1) = (67.1,68.8)

# Figure 5.1 Equilibrium in the Centralized Model

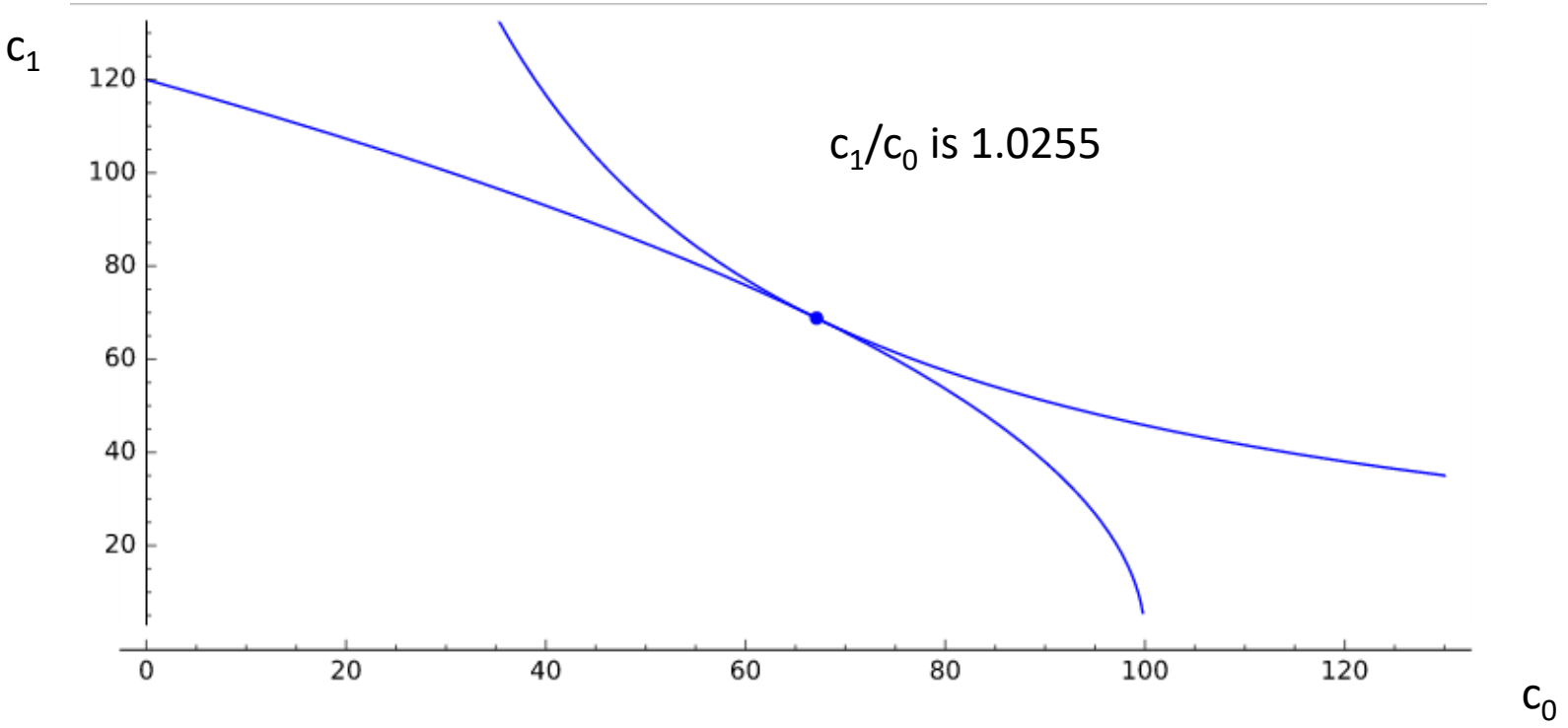



plot(((e^8.3532)/c0)^(1/.98), 0, 130, ymax=130)+plot(12*(100-c0)^.5, 0, 130)+point((67.114,68.816), size=30)

# Figure 5.6 is same as Figure 5.1 with a budget line.

- Use the budget constraint to generate the budget line as $c_1 = a + b\, c_0$
- Note that $k^s = y_0 - c_0$ from period 0
- $c_1 = \Pi + (1+r)\, k^s$ , and $1+r = 1.0463$ from decentralized model

# Figure 5.1 Equilibrium in the Centralized Model




plot(((e^8.3532)/c0)^(1/.98), 0, 130, ymax=130)+plot(12*(100-c0)^.5, 0, 130)+point((67.114,68.816), size=30)

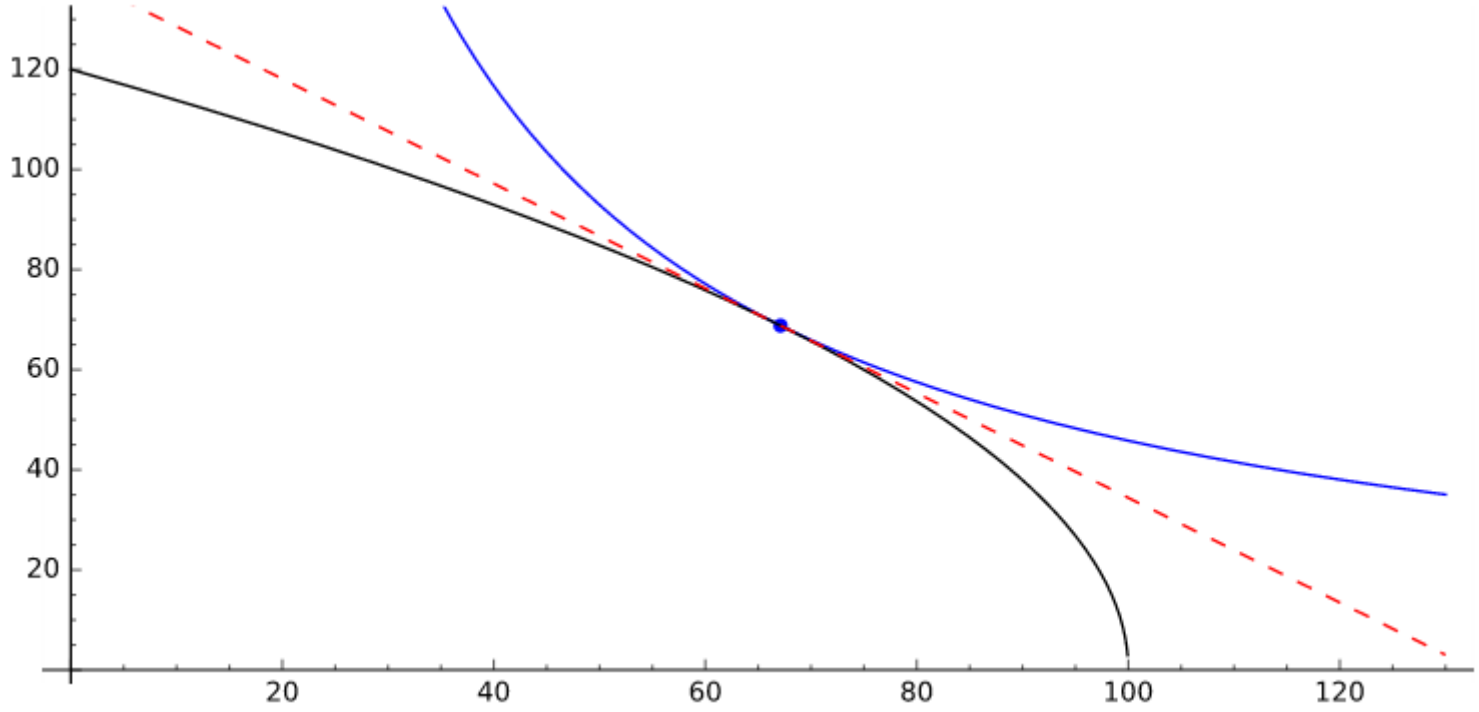
# Assume that $A_G$ doubles in the baseline model

- How do the equilibrium graphs change?
- What happens utility?
- What happens to the capital stock?
  - What happens to $c_0$?
  - What happens to $c_1$?

- What is the general expression for the capital stock, that is, before calibration?

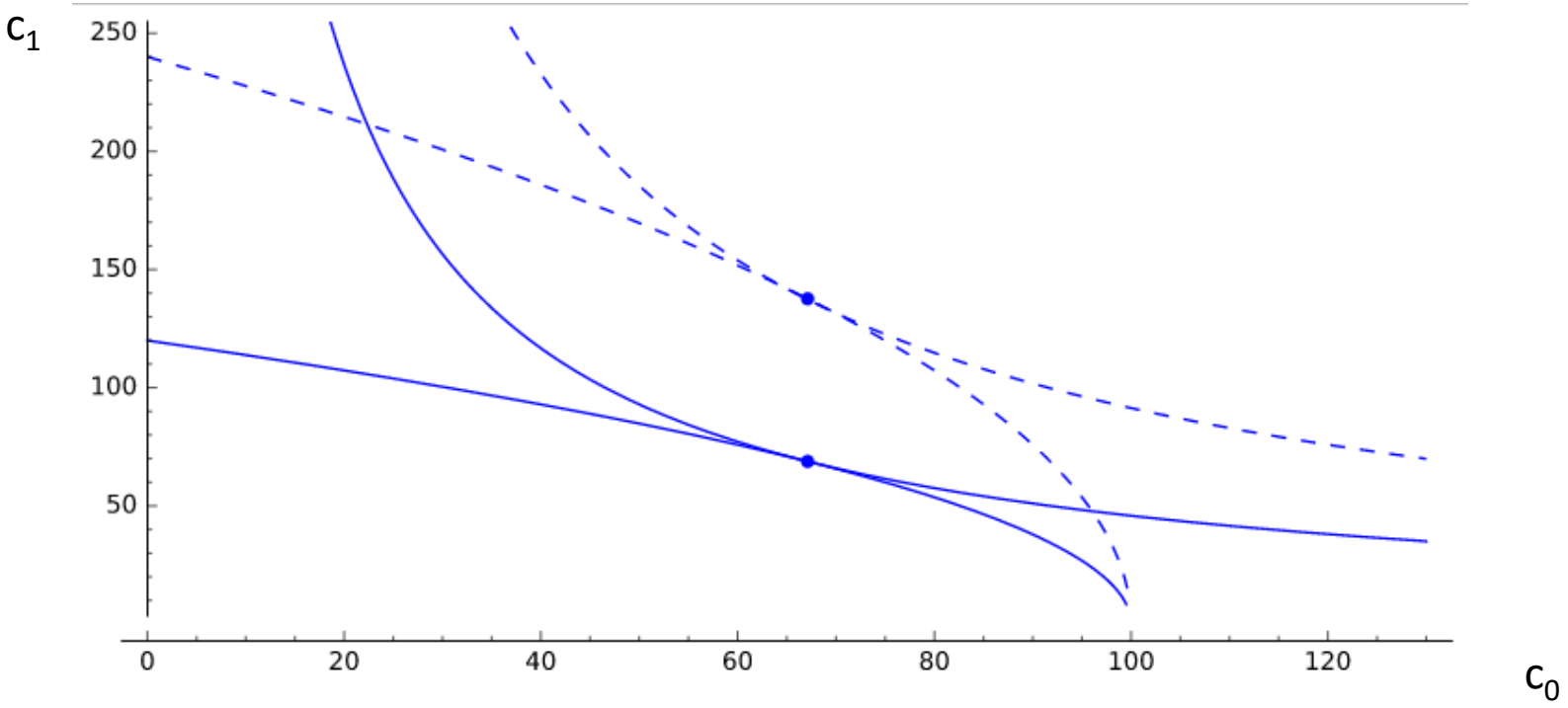
$$k_1 = y_0 \beta(1-\gamma)/(1+ \beta(1-\gamma))$$

# Figure 5.6 Equilibrium in the Decentralized Model




plot(((e^8.3532)/c0)^(1/.98), 0, 130, ymax=130)+plot(12*(100-c0)^.5, 0, 130, color='black')+point((67.114,68.816), size=30)+plot(-1.0463*c0 +1.0463*67.114+68.816, 0, 130, ymin=0, linestyle='--', color='red' )

# Figure 5.2 Equilibrium when productivity doubles

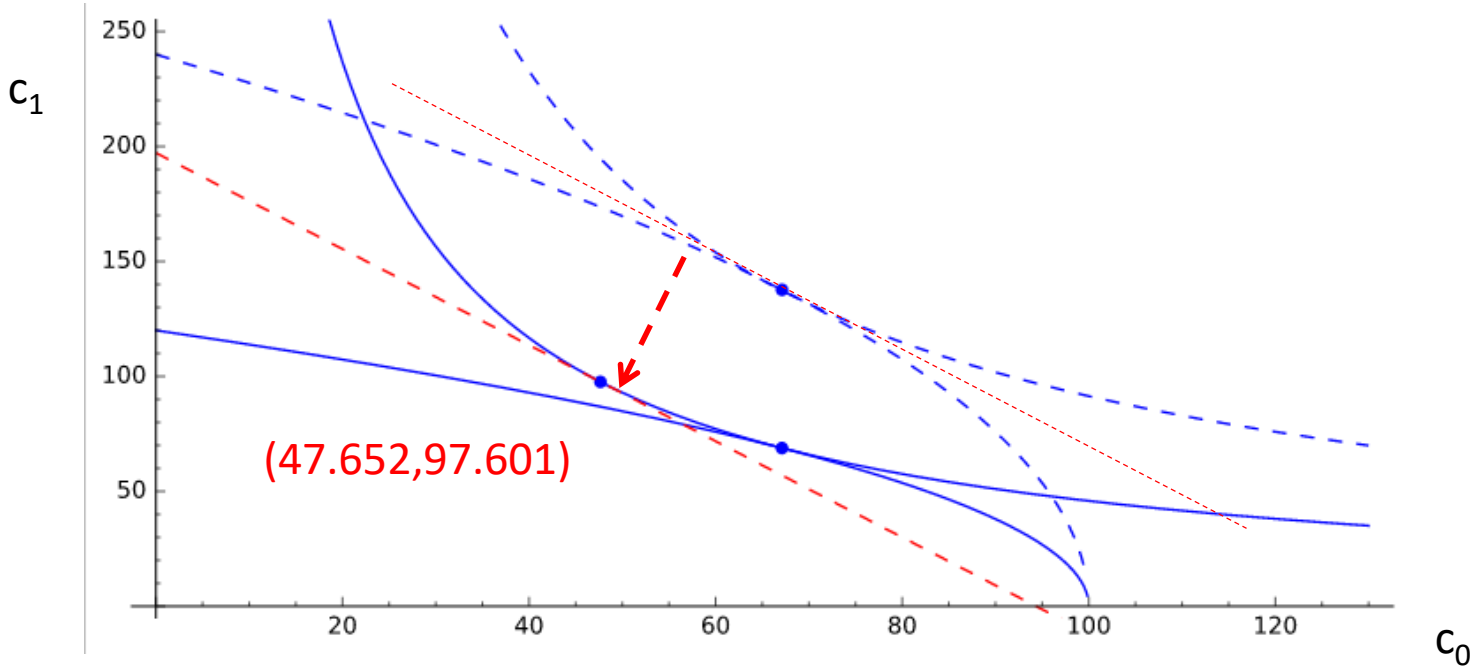



plot(((e^8.3532)/c0)^(1/.98), 0, 130, ymax=130)+plot(12*(100-c0)^.5, 0, 130)+point((67.114,68.816), size=30)+plot(((e^9.03)/c0)^(1/.98), 0, 130, ymax=250, linestyle='--')+plot(24*(100-c0)^.5, 0, 130, linestyle='--')+point((67.114,137.64), size=30)

# Compute Substitution Effect (pp 216-217)

- Find the point on A's utility that has the slope of B's utility in equilibrium for B.
- $y_0 = 100$, $\beta = .98$,
- You need 2 equations in 2 unknowns ($c_0$, $c_1$)
  - MRS for B: $(\beta/c_1)/(1/c_0) = 2.09$
  - Utility of A is the $\log c_0 + \beta \log c_1$
- At point of substitution effect MRS for A is 2.09

# Figure 5.3
# Substitution and Income Effects

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plot(((e^8.3532)/c0)^(1/.98), 0, 130, ymax=130)+plot(12*(100-c0)^.5, 0,
130)+point((67.114,68.816), size=30)+plot(((e^9.03)/c0)^(1/.98), 0, 130,
ymax=250, linestyle='--')+plot(24*(100-c0)^.5, 0, 130, linestyle='--
')+point((67.114,137.64), size=30)+point((47.652,97.601), size=30)+plot(-
2.09*c0+2.09*47.652+97.601, 0, 130, ymin=0, linestyle='--', color='red' )
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# Homework for Feb 27

- Read Chapter 6 (pp 250 to 275)
- Replicate figures 6.2, 6.4, 6.7, and 6.8
- Read Chapter 7 in preparation for class on Mar 1.
- See revised syllabus (Feb 22)