Chapters 12: Human Capital and Endogenous Growth (pp. 496-543)

April 5, 2017

Homework for April 5

- Read Chapter 12

- Chapter 12:
- Extends dynamic exogenous growth
  - - human capital investment sector,
  - - leading to endogenous growth. ·
- Human capital enters goods production, -
  - with its growth comparable to productivity growth. -
  - Instead of goods productivity rising exogenously.
- Separate new sector, adds 2nd intertemporal margin
  - through this growth rate endogenous.
  - BGP returns on human, physical capital equal.
  - State variable: ratio of physical to human capital.
- AS-AD presented with modifications:
  - account for time in human capital production,
  - human capital productivity parameter.

Solution Methodology

- AS-AD used to get single equation in one unknown.
- Complication: a quadratic equation, two roots.
  Rule out economically infeasible solution.
  Solution methodology in Appendix A12.
- Here, solve quadratic for g, growth rate instead of capital stock, since calibration targets a growth rate value

What is meant by “two sector” economy in this chapter?

- Note the *y* output can be used interchangeably as either *c* or *i*, investment in physical capital. It cannot be used as investment in human capital, i_h.
- Two sectors means two kinds of output, *y* and i_h, each with its own production function.

Human Capital Investment

- Human capital, h_t, with h_t=1, and
  - $w_t h_t$ effective wage rate;
  - $l_t h_t$ "effective labor time".
  - Time augmented by human capital.
- w_t wage of "raw" labor
  - stable along growth path instead of rising as with exogenous growth.
  - Per capita income rises only because human capital rises.
- Goods production function with *y* growing at rate *g*

$$y_t^s = A_G \left(l_t^d h_t \right)^{\gamma} (k_t)^{1-\gamma}$$

See Relation to Chapter 11, Exogenous Growth

$$y_t^s = \left(l_t^d h_t\right)^{\gamma} (k_t)^{1-\gamma},$$

$$h_t = (A_{Gt})^{\frac{1}{\gamma}},$$

$$y_t^s = A_{Gt}\left(l_t^d\right)^{\gamma} (k_t)^{1-\gamma}.$$

Production of human capital is linear in labor

$$i_{ht} = A_H l_{Ht} h_t.$$

$$1 = l_{Ht} + l_t^s + x_t.$$

$$T_t \equiv l_t^s + x_t = 1 - l_{Ht},$$

$$1 = T_t + l_{Ht}.$$

$$h_{t+1} = h_t(1 - \delta_h) + i_{ht},$$

What is the margin of tradeoff for the household?

Set up the model with endogenous growth and take FOCs and Envelope Conditions.

$$c_t^d = w_t l_t^s h_t + r_t k_t - k_{t+1} + k_t - \delta_k k_t.$$

$$V(k_t, h_t) = \text{ Maximum value of}$$

$$\ln[w_t l_t^s h_t + r_t k_t - k_{t+1} + k_t - \delta_k k_t] + \alpha \ln(x_t) + \beta V(k_{t+1}, h_{t+1}),$$

where

$$h_{t+1} = h_t(1 - \delta_h) + A_H l_{Ht} h_t,$$

$$x_t = 1 - l_{Ht} - l_t^s.$$

$$V(k_t, h_t) = \text{ Maximum value of}$$

$$\ln[w_t l_t^s h_t + r_t k_t - k_{t+1} + k_t - \delta_k k_t]$$

$$+ \alpha \ln(1 - l_{Ht} - l_t^s) + \beta V[k_{t+1}, h_t(1 - \delta_h) + A_H l_{Ht} h_t].$$

Solve for the three margins, one intratemporal and two intertemporal.

First Order Conditions for capital in the nextand period 2 uses of labor

$$k_{t+1} : \frac{1}{c_t^d}(-1) + \beta \frac{\partial V(k_{t+1}, h_{t+1})}{\partial k_{t+1}} = 0,$$

$$l_t^s : \frac{1}{c_t^d}(w_t h_t) + \frac{\alpha}{x_t}(-1) = 0,$$

$$l_{Ht} : \frac{\alpha}{x_t}(-1) + \beta \frac{\partial V(k_{t+1}, h_{t+1})}{\partial h_{t+1}}(A_H h_t) = 0;$$

Envelope Conditions

$$k_t : \frac{\partial V(k_t, h_t)}{\partial k_t} = \frac{1}{c_t^d}(1 + r_t - \delta_k),$$

$$h_t : \frac{\partial V(k_t, h_t)}{\partial h_t} = \frac{1}{c_t^d}(w_t l_t) + \beta \frac{\partial V(k_{t+1}, h_{t+1})}{\partial h_{t+1}}(1 + A_H l_{Ht} - \delta_H).$$

What are the marginal conditions that determine the equilibrium values?

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$$MRS_{c_t,c_{t+1}} : 1 + g_t = \frac{1 + r_t - \delta_k}{1 + \rho};$$

$$MRS_{x_t,c_t} : \frac{\alpha c_t^d}{x_t} = w_t h_t = MP_{l_t}$$

What is the intertemporal margin for human capital?

What is the intertemporal margin for human capital?

$$\frac{\partial V(k_{t+1},h_{t+1})}{\partial h_{t+1}} = \frac{\alpha}{\beta(A_H h_t)x_t}; \quad \frac{\partial V(k_t,h_t)}{\partial h_t} = \frac{\alpha}{\beta(A_H h_{t-1})x_{t-1}},$$

$$\frac{\partial V(k_t,h_t)}{\partial h_t} = \frac{1}{c_t^d}(w_t l_t) + \beta\frac{\partial V(k_{t+1},h_{t+1})}{\partial h_{t+1}}(1 + A_H l_{Ht} - \delta_H);$$

$$\frac{(h_t)x_t}{(h_{t-1})x_{t-1}} = \beta(A_H)l_t^s + \beta(1 + A_H l_{Ht} - \delta_H).$$

$$1 + g_t = \frac{1 + A_H(1 - x_t) - \delta_h}{1 + \rho} = \frac{1 + r_t - \delta_k}{1 + \rho};$$

$$\Rightarrow r_t - \delta_k = A_H(1 - x_t) - \delta_h.$$

Growth rate g depends on

1. A_H human capital investment productivity
2. time productively employed, 1-x,
3. marginal product of physical capital r_t.

- Links directly: employment rate and growth rate.

AS-AD with Human Capital

$$
\begin{aligned}
c_t^d &= w_t l_t^s h_t + r_t k_t - [k_t(1+g) - k_t(1-\delta_k)] \\
&= w_t l_t^s h_t + k_t(r_t - \delta_k - g).
\end{aligned}
$$

$$
\frac{c_t \alpha}{w_t h_t} = x_t, \; x_t = 1 - l_t^s - l_{Ht}; \quad \Rightarrow l_t^s = 1 - \frac{c_t^d \alpha}{w_t h_t} - l_{Ht}.
$$

$$
c_t^d = w_t h_t \left(1 - \frac{c_t^d \alpha}{w_t h_t} - l_{Ht} \right) + k_t(r_t - \delta_k - g),
$$

$$
c_t^d = \frac{1}{1+\alpha}[w_t h_t(1 - l_{Ht}) + k_t(r_t - \delta_k - g)];
$$

$$
1 + g = \frac{1 + r - \delta_k}{1 + \rho}; \quad \Rightarrow r_t - \delta_k - g = \rho(1+g),
$$

$$
c_t^d = \frac{1}{1+\alpha}[w_t h_t(1 - l_{Ht}) + k_t \rho(1+g)].
$$

Aggregate Supply

$$l_t^d = \left(\frac{\gamma A_G}{w_t}\right)^{\frac{1}{1-\gamma}} \frac{k_t}{h_t}.$$

$$y_t^s = A_G\left(l_t^d h_t\right)^{\gamma}(k_t)^{1-\gamma},$$

$$y_t^s = A_G\left(\frac{\gamma A_G}{w_t}\right)^{\frac{\gamma}{1-\gamma}} k_t,$$

$$\frac{y_t^s}{h_t} = A_G\left(\frac{\gamma A_G}{w_t}\right)^{\frac{\gamma}{1-\gamma}} \frac{k_t}{h_t}.$$

$$\frac{1}{w_t} = \frac{1}{\gamma A_G}\left(\frac{y_t^s}{A_G k_t}\right)^{\frac{1-\gamma}{\gamma}}.$$

It is exactly the same AS function as in Chapter 8 when there is exogenous growth

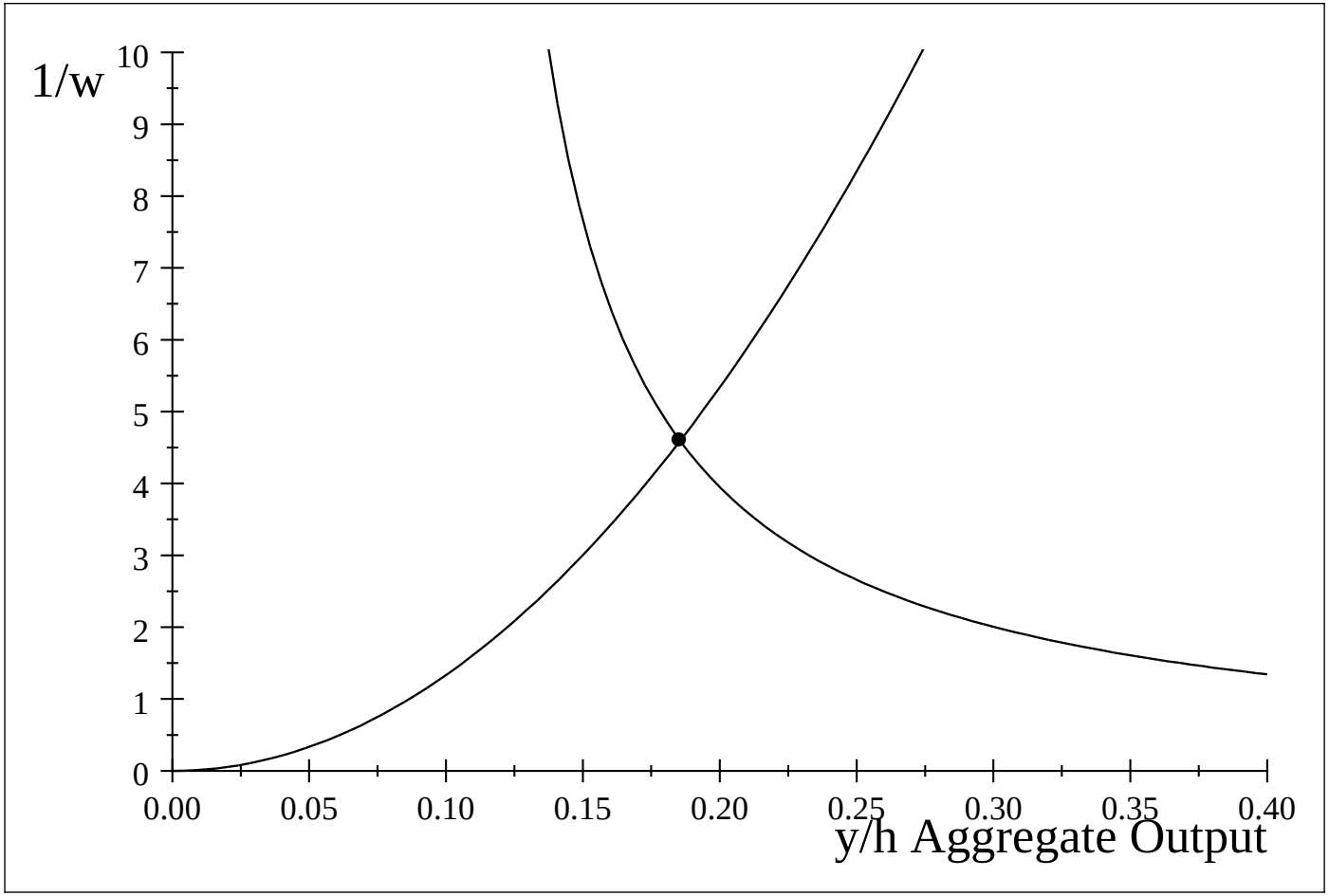
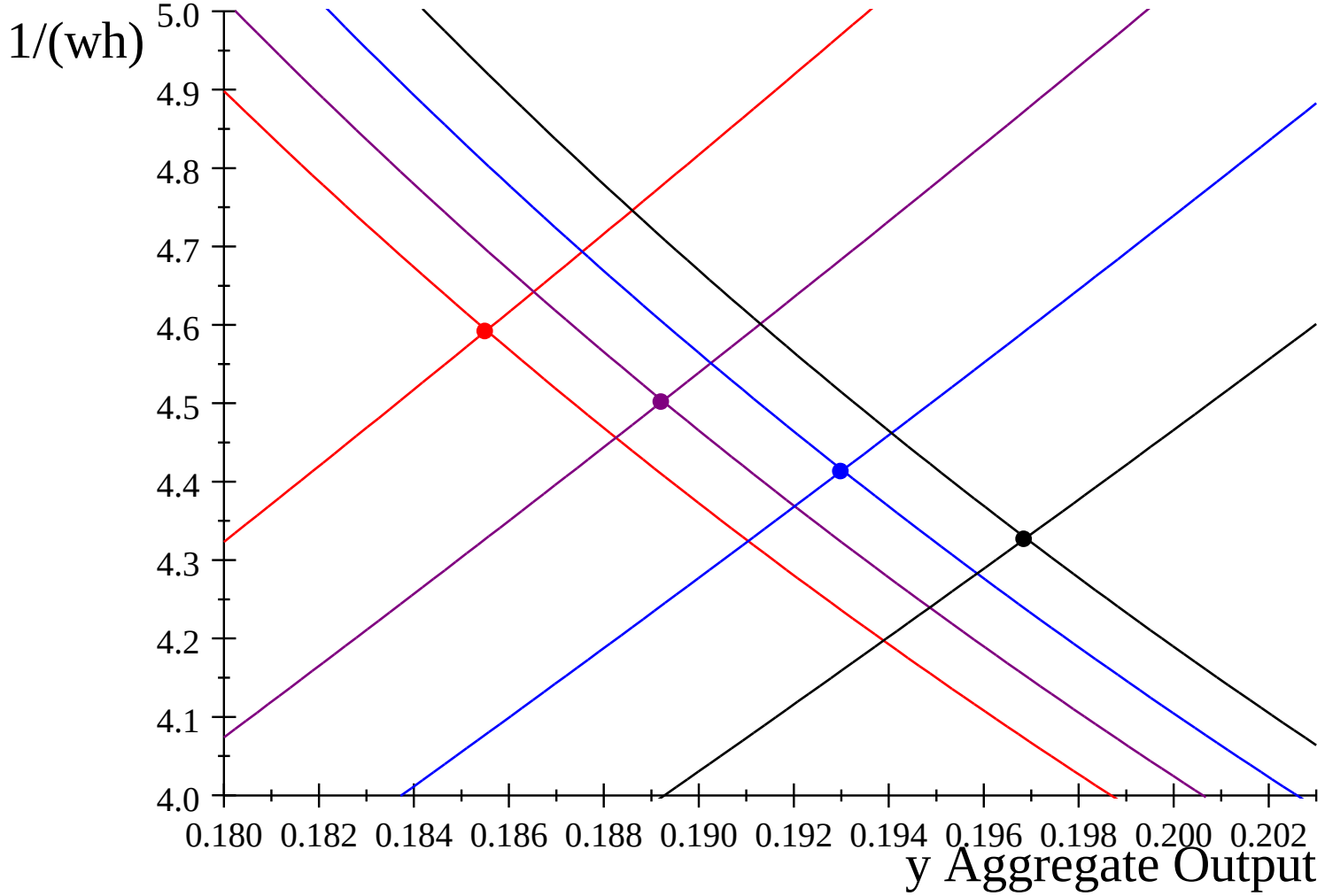


Figure 12. 1. *AS – AD* with Human Capital and Endogenous Growth

Figure 12.7. Endogenous Growth shifts in AS-AD from time t to t+3



Homework for April 24

- Finish Growth Homework (Chapter 11 and 12) and Quizzes 19 and 20.
- Read Chapter 17
- Quiz 21

1. Set up and solve the model with bonds (Sections 17.2.1 and 17.2.2
2. What is the Expectations Hypothesis of Asset Prices (Section 17.4.2)
3. Explain in words what you think is the Expectations Hypothesis of Interest Rates.