Chapters 11: Exogenous Growth (pp 454-485)

April 3, 2017

Homework for April 3

- Review Midterm Results
- Read Chapter 11 (Note Typo in Section 11.4.1 Should be $w_t T$, not $T w_t l_t$)
- Derive equations for AD-AS and replicate Figures 11.1 and 11.4. If you use R, include the programming.

Quiz 19 Chapter 11 April 3, 2017

1. How do you add exogenous growth to the baseline model of chapter 8?
2. What is a ‘Stylized Fact’?
3. Show with hand drawing
   a. How the baseline AD-AS changes when you add 2% balanced growth to the baseline model?
   b. How the AD-AS figure with 2% growth changes if you lower the rate of time preference enough so that the return to capital stays at the baseline value of 6%.
   c. How the baseline LD-LS changes when you add 2% balanced growth to the baseline model?
4. What is growth accounting?

1. With Exogenous Technological Progress

$$y_t = A_{Gt}(l_t)^{\gamma}(k_t)^{1-\gamma}.$$

$$A_{Gt+1} = A_{Gt}(1+\mu),$$

Re-write production function with A_{Gt} factoring labor time :

$$y_t = \left[l_t (A_{Gt})^{\frac{1}{\gamma}} \right]^{\gamma} (k_t)^{1-\gamma}.$$

Define $\tilde{A}_{Gt} \equiv (A_{Gt})^{\frac{1}{\gamma}}$, then

$$y_t = [l_t \tilde{A}_{Gt}]^{\gamma} (k_t)^{1-\gamma}.$$

What is a stylized fact?

- Model building guided by data.
  - Is it a fact or something else?
  - **Stylized facts** are facts represented in a way that simplifies details rather than trying to show naturalness or reality.
  - **Empirical Regularities** are properties of data generated by government agencies and academic researchers. These properties are evident over time and found across a wide variety of countries.
  - I never would use the term stylized facts to describe the empirical guidelines used to build models. Although I appear to be fighting a losing battle as the usage has become common.

Empirical Regularities in U.S. Economic Growth

Real interest rate remains constant.

Hours worked per person stay relatively constant

Great ratios remain relatively constant

- Output to capital ratio
- Consumption to output
- Investment to output

- Real wage rises over time at the same rate as labor productivity.
- Per capita income rises along the balanced growth path.
- Output, consumption and investment all tend to grow along the same balanced growth path.

Implications of the baseline model with exogenous growth in total factor productivity.

$$\frac{y_t}{k_t} = \left[\frac{l_t \tilde{A}_{Gt}}{k_t} \right]^{\gamma},$$

$$r_t = MP_{k_t} = (1-\gamma)\frac{y_t}{k_t} = (1-\gamma)\left[\frac{l_t \tilde{A}_{Gt}}{k_t} \right]^{\gamma}$$

$$\Rightarrow \Delta\left(\frac{l_t \tilde{A}_{Gt}}{k_t} \right) = 0; \;\; \Delta(l_t) = 0, \Rightarrow \Delta(\tilde{A}_{Gt}) = \Delta k_t$$

$$1+g = \frac{y_{t+1}}{y_t} = \frac{\left[l_{t+1}(A_{Gt+1})^{\frac{1}{\gamma}} \right]^{\gamma} (k_{t+1})^{1-\gamma}}{\left[l_t (A_{Gt})^{\frac{1}{\gamma}} \right]^{\gamma} (k_t)^{1-\gamma}} = (1+\mu)\left(\frac{k_{t+1}}{k_t} \right)^{1-\gamma}$$

Implications of the baseline model with exogenous growth in total factor productivity.

$$
\begin{aligned}
\frac{k_{t+1}}{k_t} &= \left(\frac{1+g}{1+\mu}\right)^{\frac{1}{1-\gamma}} \\
\left(\frac{A_{Gt+1}}{A_{Gt}}\right)^{\frac{1}{\gamma}} &= (1+\mu)^{\frac{1}{\gamma}}. \\
&\Rightarrow \left(\frac{1+g}{1+\mu}\right)^{\frac{1}{1-\gamma}} = (1+\mu)^{\frac{1}{\gamma}} \\
1+g &= (1+\mu)^{\frac{1}{\gamma}} \\
&\Rightarrow g = \frac{y_{t+1}}{y_t} = \frac{k_{t+1}}{k_t} \simeq \frac{\mu}{\gamma}.
\end{aligned}
$$

Growth Accounting and the BGP

$$g = \mu + (1-\gamma)\frac{k_{t+1}}{k_t} = \mu + (1-\gamma)\frac{\mu}{\gamma}$$

$$\frac{y_{t+1}}{y_t} = \frac{c_{t+1}}{c_t} = \frac{k_{t+1}}{k_t} = \frac{i_{t+1}}{i_t} = 1+g.$$

5. Set up and solve the GE recursive model with exogenous growth.

a. Write down the Bellman equation, the utility function, the four constraints.
b. Write down the modified Bellman equation for the decentralized solution.
c. Take first order conditions and envelope condition and solve for the intertemporal and intratemporal equilibrium conditions.

a. Model same as chapter 8 plus modification to A_{Gt}

$$V(k_t) = \underset{c_t, x_t, l_t, k_{t+1}}{Max} : u(c_t, x_t) + \beta V(k_{t+1}),$$

$$\text{where } u(c_t, x_t) = \ln c_t + \alpha \ln x_t.$$

$$y_t = A_{Gt} l_t^{\gamma} k_t^{1-\gamma}, \text{where } A_{Gt+1} = (1+\mu) A_{Gt}$$

$$y_t = c_t + i_t.$$

$$i_t = k_{t+1} - k_t(1 - \delta_k)$$

$$T = x_t + l_t.$$

b. Write down the modified Bellman equation for the decentralized solution.

$$V(k_t^s) =$$

$$\underset{l_t^s, k_{t+1}^s}{Max}: u[w_t l_t^s + r_t k_t^s + \Pi_t - k_{t+1}^s + k_t^s(1 - \delta_k), T - l_t] + \beta V(k_{t+1}^s)$$

$$\underset{l_t^d, k_t^d}{Max} \quad \Pi_t = A_{Gt}\left(l_t^d\right)^{\gamma}\left(k_t^d\right)^{1-\gamma} - w_t l_t^d - r_t k_t^d$$

c. Take first order conditions and envelope condition and solve for the intertemporal and intratemporal equilibrium conditions.

$$0 = \frac{\partial u\left(c_t^d, x_t\right)}{\partial c_t} w_t - \frac{\partial u\left(c_t^d, x_t\right)}{\partial x_t};$$

$$0 = \frac{-\partial u\left(c_t^d, x_t\right)}{\partial c_t^d} + \beta \frac{\partial V(k_{t+1}^s)}{\partial k_{t+1}^s}.$$

$$Envelope : \frac{\partial V(k_t^s)}{\partial k_t^s} = \frac{\partial u\left(c_t^d, x_t\right)}{\partial c_t^d}(1 + r_t - \delta_k).$$

$$\Rightarrow 1 + g = \frac{1 + r_{t+1} - \delta_k}{1 + \rho}, \text{Intertemporal Margin}$$

$$w_t = \frac{\frac{\alpha}{x_t}}{\frac{1}{c_t^d}}. \text{Intratemporal Margin}$$

See formula for *k*, with and without growth

Without growth

$$\frac{T\gamma A_G^{\frac{1}{\gamma}}\left[\frac{(1-\gamma)}{\rho+\delta_k}\right]^{\frac{1-\gamma}{\gamma}}}{(\gamma+\alpha)\left(\frac{\rho+\delta_k}{(1-\gamma)}\right)-\alpha\delta_k}$$

With growth

$$\frac{T\gamma(A_G)^{\frac{1}{\gamma}}\left(\frac{(1-\gamma)}{g(1+\rho)+\rho+\delta_k}\right)^{\frac{1-\gamma}{\gamma}}}{(1+\alpha)\delta_k\left(\frac{\gamma}{(1-\gamma)}\right)+(1+\alpha)\left(\frac{g(1+\rho)+\rho}{(1-\gamma)}\right)-[\rho(1+g)+(1+\alpha)g]}$$

Figure 11.4 AS-AD Equilibria with 2% growth

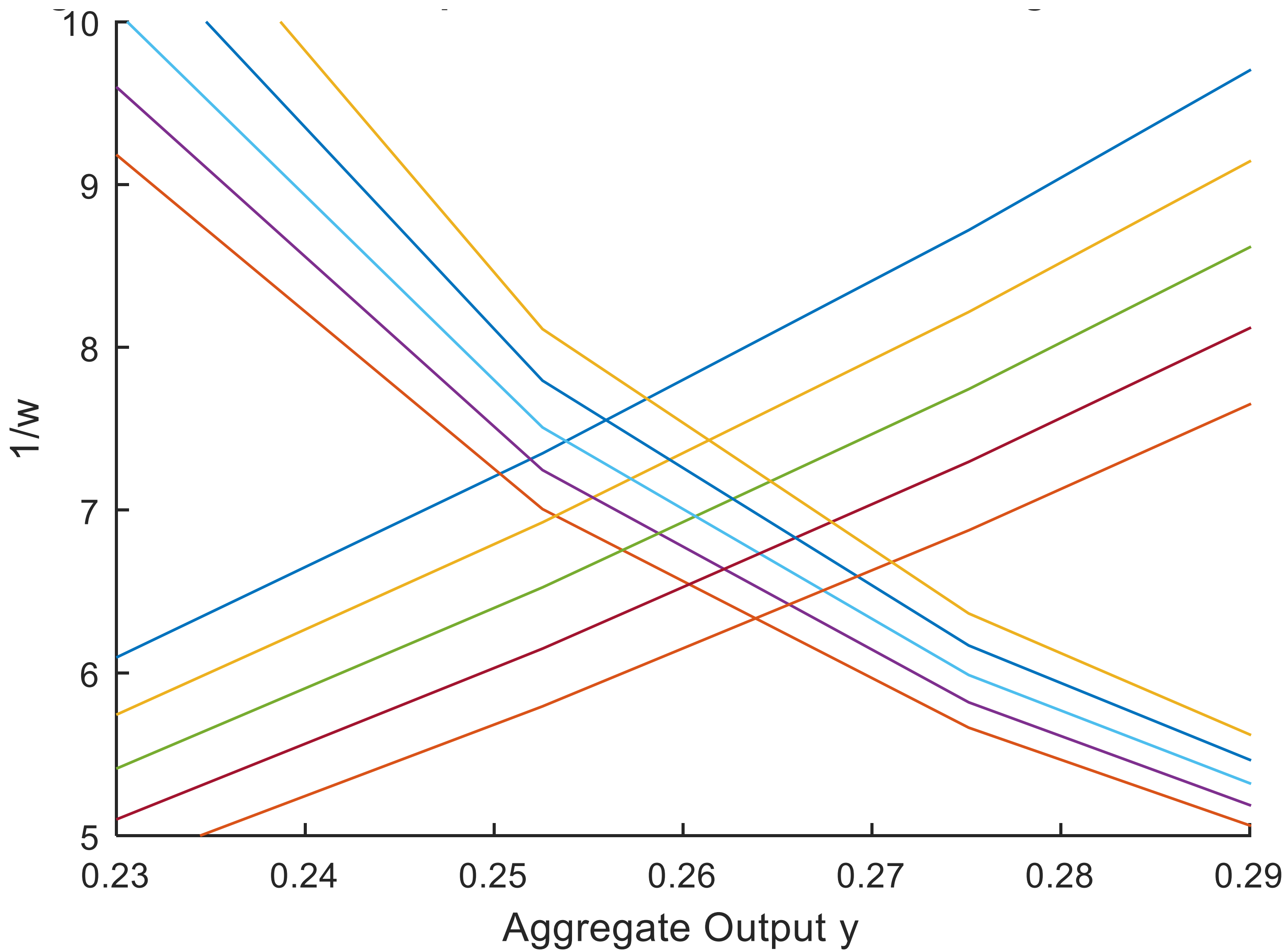


Figure 11.5 Labor Market 7 periods later

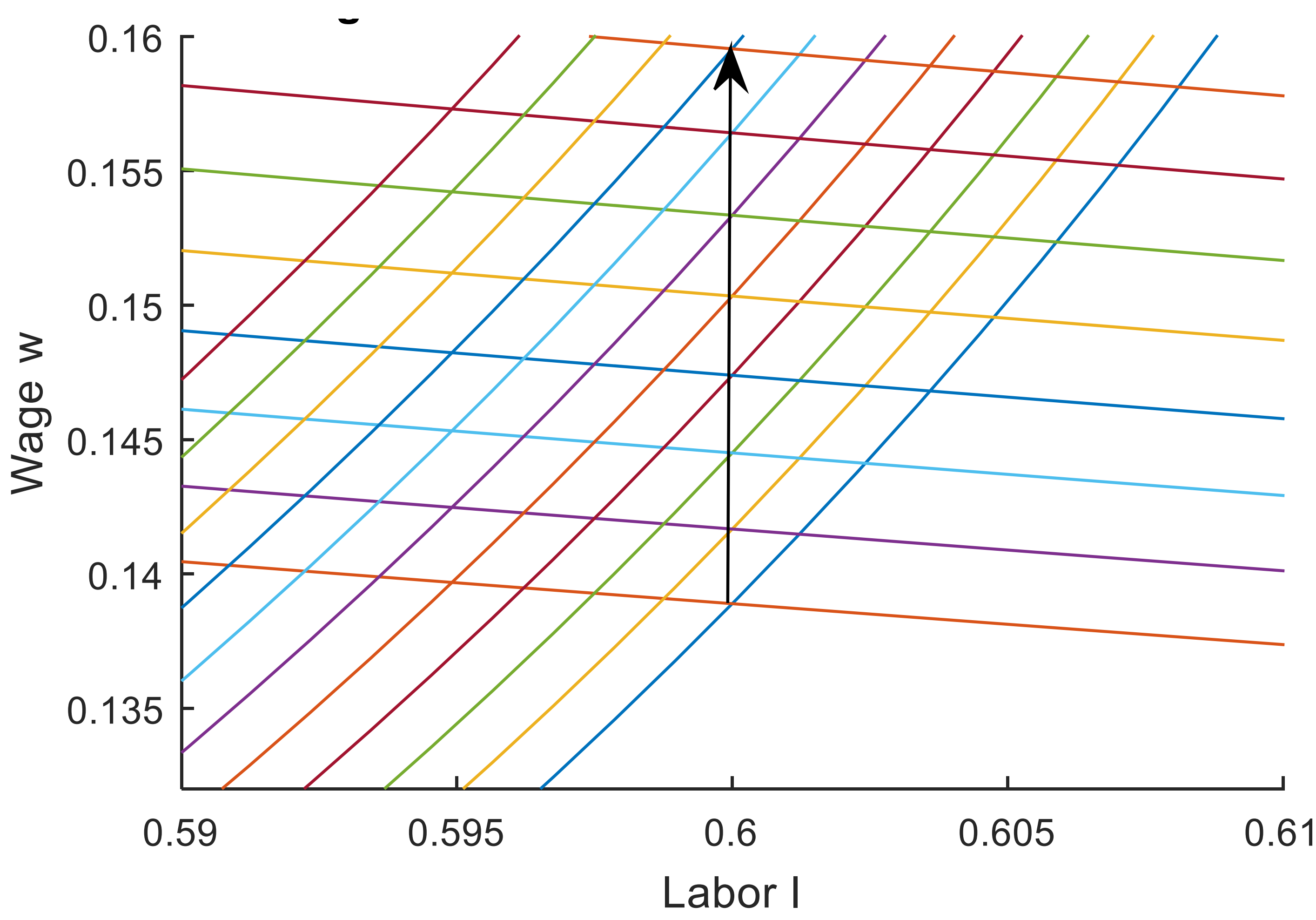


Figure 11.6 Factor Market Equilibrium 3 periods later

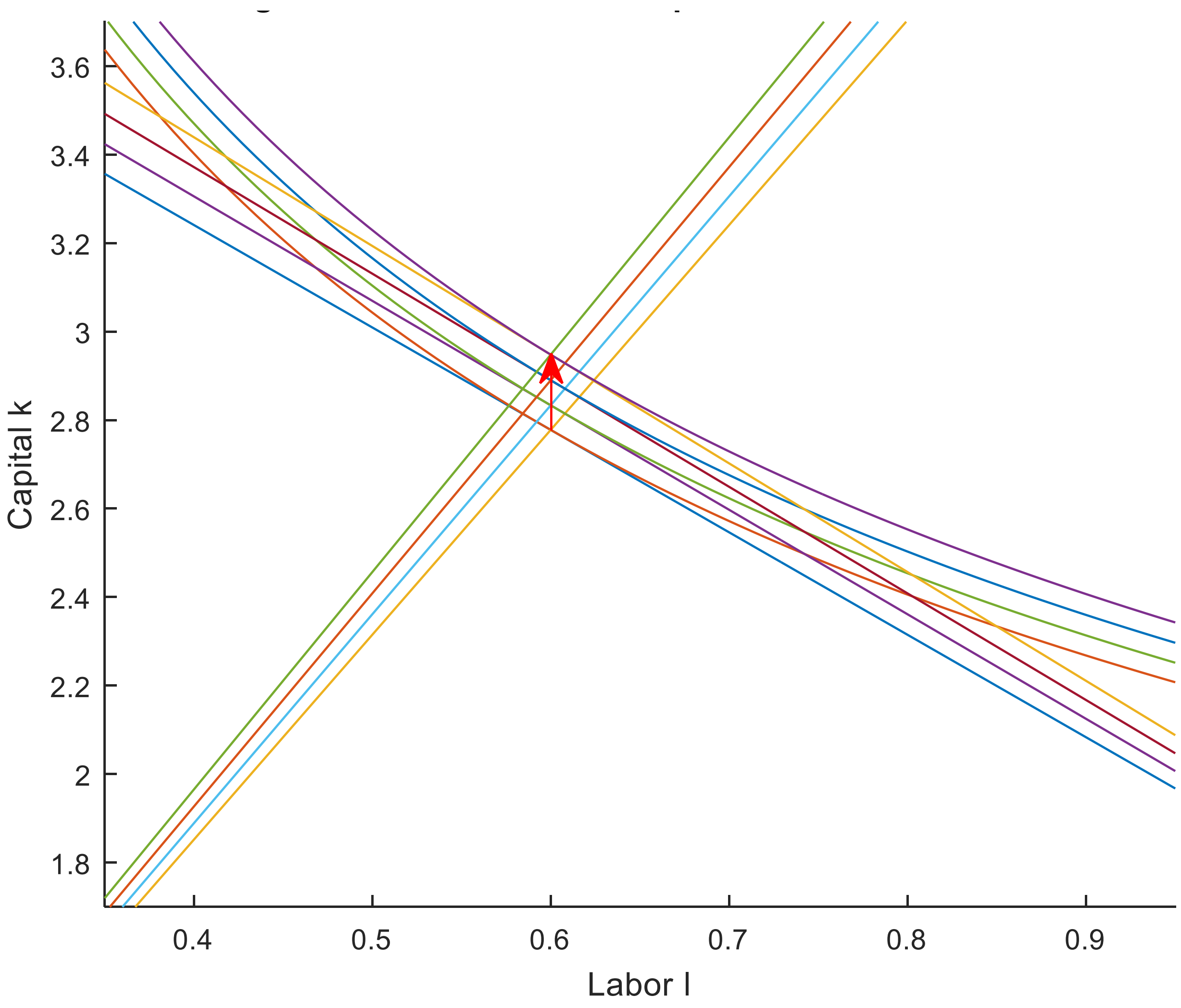


Figure 11.7 General Equilibruim--Production (c) Utility and Labor

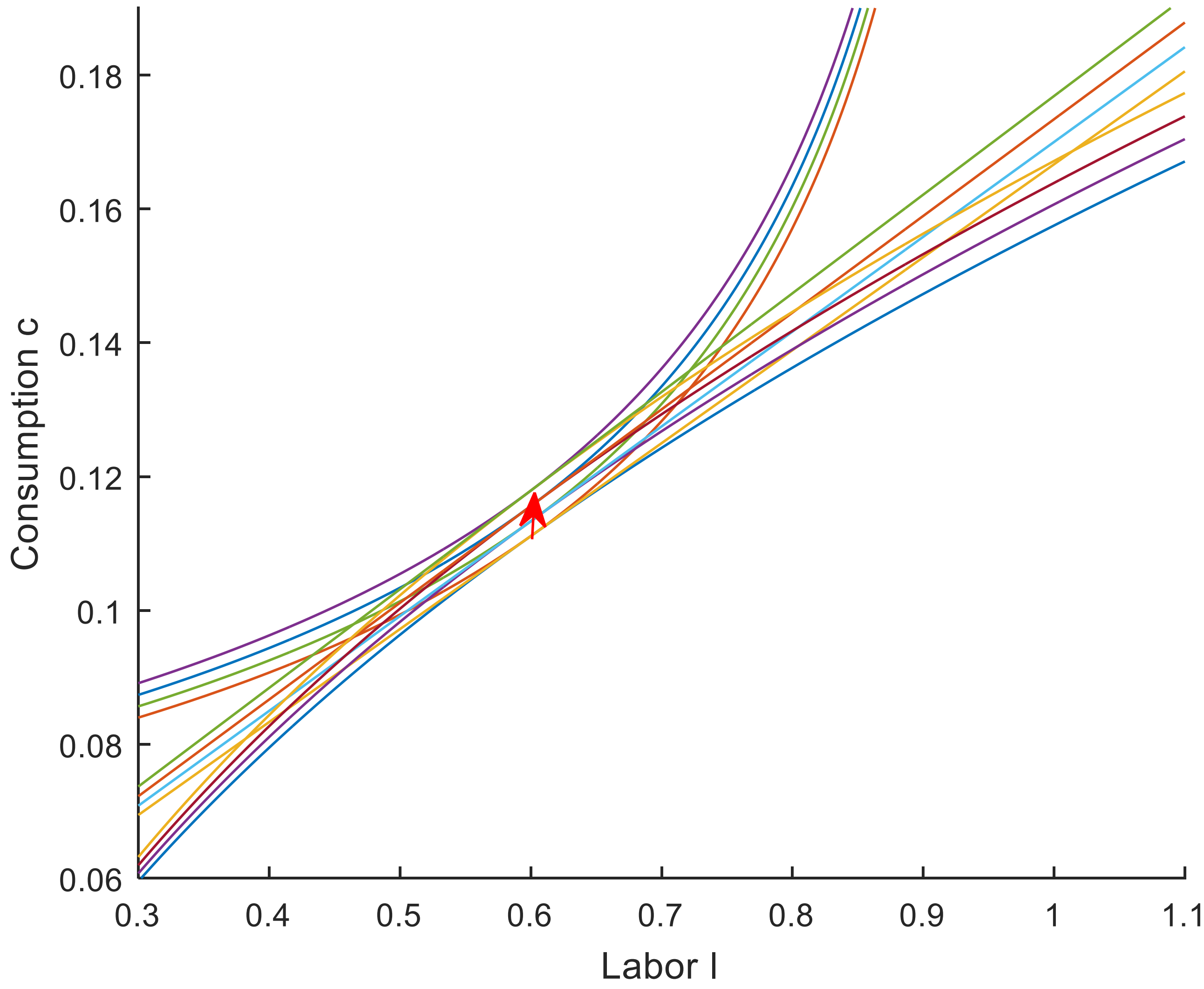


Figure 11.8 AS-AD Equilibrium with Time going down and Ag going up

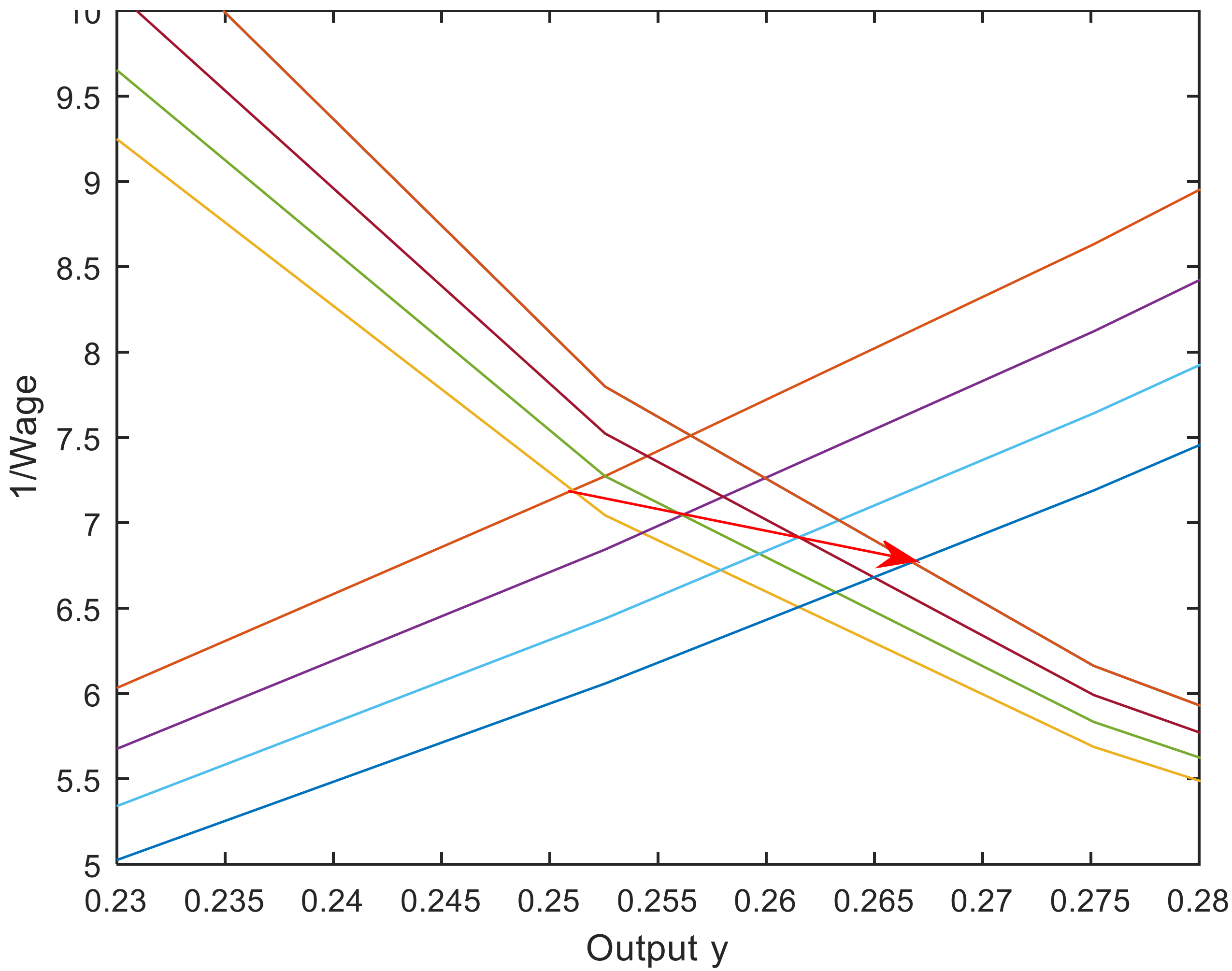
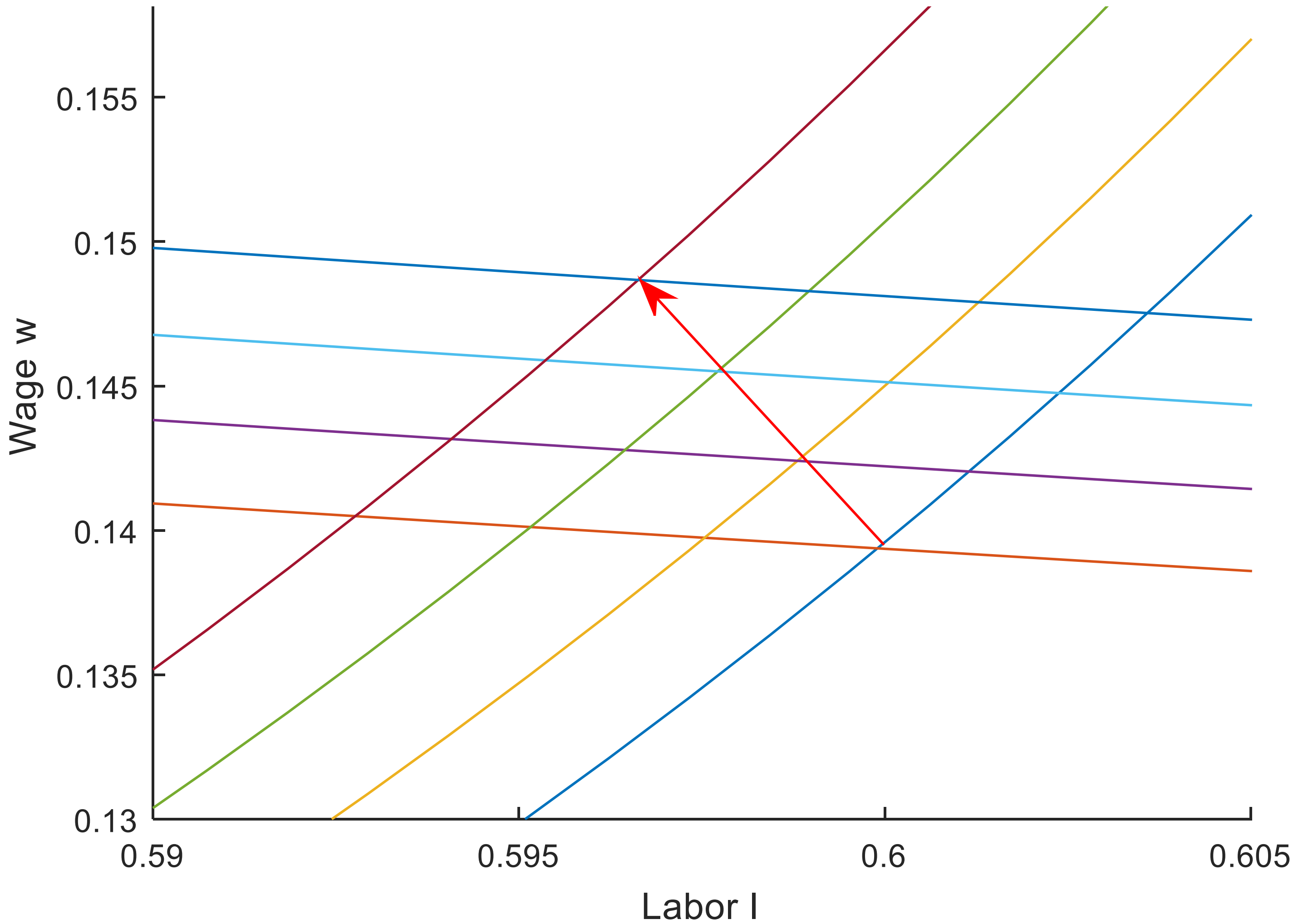


Figure 11.9 Labor Market with T going down and Ag going up



Homework for April 5

- Read Chapter 12