Chapters 17: Asset Prices and Finance (pp. 743-760)

April 26, 2017

Homework for April 26

- Finish reading chapter 17, from section 17.4 to the end (pages 743-760)
- Quiz 22

1. How does the introduction of uncertainty complicate our solutions?
2. What is the Equity Premium?
3. What is the Expectations Hypothesis of Asset Prices (Section 17.4.2)?
4. Explain how adding human capital to the model might explain the Equity Premium Puzzle.

Set up the Model with Asset Pricing under Uncertainty—FOCs for b_{t+1} and s_{t+1}

$$V(b_t, s_t) = \underset{l_t, b_{t+1}, s_{t+1}}{Max}$$

$$E_t\{\ln[w_t l_t - b_{t+1} + b_t(1+R_t) - p_t^s s_{t+1} + (p_t^s + d_t^s)s_t]$$

$$+\alpha \ln(1-l_t) + \beta V(b_{t+1}, s_{t+1})\};$$

Write out the First Order Conditions:

Set up the Model with Asset Pricing under Uncertainty—FOCs for b_{t+1} and s_{t+1}

$$V(b_t, s_t) = \underset{l_t, b_{t+1}, s_{t+1}}{Max}$$

$$E_t\{\ln[w_t l_t - b_{t+1} + b_t(1+R_t) - p_t^s s_{t+1} + (p_t^s + d_t^s) s_t]$$

$$+\alpha \ln(1 - l_t) + \beta V(b_{t+1}, s_{t+1})\};$$

$$w_t = \frac{\alpha c_t}{x_t}, \quad \frac{\partial u(c_t, x_t)}{\partial c_t} = \beta E_t\left[\frac{\partial V(b_{t+1}, s_{t+1})}{\partial b_{t+1}}\right],$$

$$p_t^S \frac{\partial u(c_t, x_t)}{\partial c_t} = \beta E_t\left[\frac{\partial V(b_{t+1}, s_{t+1})}{\partial s_{t+1}}\right],$$

Write out the envelope conditions:

Envelope Conditions for b_t and s_t

$$\frac{\partial V(b_t, s_t)}{\partial b_t} = \frac{\partial u(c_t, x_t)}{\partial c_t}(1 + R_t),$$

$$\frac{\partial V(b_t, s_t)}{\partial s_t} = \frac{\partial u(c_t, x_t)}{\partial c_t}(p_t^s + d_t^s).$$

Solve for the equilibrium condition for asset pricing.

Basic Asset Pricing Condition

$$p_t^S = E_t\left[\frac{\beta \frac{\partial u(c_{t+1}, x_{t+1})}{\partial c_{t+1}}}{\frac{\partial u(c_t, x_t)}{\partial c_t}} (p_{t+1}^S + d_{t+1}^S) \right].$$

$$\frac{1}{1 + R_{t+1}} = E_t\left[\frac{\beta \frac{\partial u(c_{t+1}, x_{t+1})}{\partial c_{t+1}}}{\frac{\partial u(c_t, x_t)}{\partial c_t}} \right].$$

$$p_t^S = E_t\left[\left(\frac{1}{1 + R_{t+1}} \right) (p_{t+1}^S + d_{t+1}^S) \right].$$

Defining risk as covariance

$$X_t \equiv \frac{1}{1+R_{t+1}},\ Y_t \equiv p_{t+1}^S + d_{t+1}^S.$$

$$p_t^S = E_t[X_t Y_t] = E_t[X_t]E_t[Y_t] + Cov_t[X_t Y_t].$$

Suppose $Cov_t[X_t Y_t] = 0$,

$$\Rightarrow p_t^S = E_t\left[\frac{1}{1+R_{t+1}}\right]E_t[p_{t+1}^S + d_{t+1}^S],$$

$$p_t^S = \frac{E_t[p_{t+1}^S + d_{t+1}^S]}{1+R_{t+1}}.$$

"Expectations Hypothesis of Asset Pricing"

Prices as Dividend Stream under Zero Covariance

$$p_t^S = \frac{E_t[p_{t+1}^S + d_{t+1}^S]}{1+R_{t+1}}, \; p_{t+1}^S = \frac{E_{t+1}(p_{t+2}^S + d_{t+2}^S)}{1+R_{t+2}};$$

$$p_t^s = \frac{E_t\left[E_{t+1}\left[\frac{\left(p_{t+2}^S + d_{t+2}^S\right)}{1+R_{t+2}} \right] + d_{t+1}^S \right]}{1+R_{t+1}}$$

$$= \frac{E_t[d_{t+1}^S]}{1+R_{t+1}} + E_t E_{t+1}\left[\frac{p_{t+2}^S + d_{t+2}^S}{(1+R_{t+1})(1+R_{t+2})} \right]$$

$$= \frac{E_t[d_{t+1}^S]}{1+R_{t+1}} + E_t E_{t+1}\left[\frac{E_{t+2}(p_{t+3}^S + d_{t+3}^S)}{(1+R_{t+1})(1+R_{t+2})(1+R_{t+3})} \right]$$

$$+ E_t E_{t+1}\left[\frac{d_{t+2}^S}{(1+R_{t+1})(1+R_{t+2})} \right] + \ldots .$$

$$\lim_{j\to\infty} \frac{p^S(k_j)}{(1+R_{t+1})(1+R_{t+2})\cdots(1+R_{t+j})} = 0.$$

and $E_t E_{t+1} E_{t+2} = E_t E_{t+1} E_{t+2} E_{t+3} = \ldots = E_t$,

$$p_t^s = \frac{E_t[d_{t+1}^S]}{1+R_{t+1}} + E_t E_{t+1}\left[\frac{d_{t+2}^S}{(1+R_{t+1})(1+R_{t+2})}\right]$$

$$+ E_t E_{t+1} E_{t+2}\left[\frac{d_{t+3}^S}{(1+R_{t+1})(1+R_{t+2})(1+R_{t+3})}\right]$$

$$+\ldots \lim_{j\to\infty} \frac{p^S(k_j)}{(1+R_{t+1})(1+R_{t+2})\cdots(1+R_{t+j})};$$

$$\Rightarrow p_t^S = \sum_{j=1}^{\infty} E_t\left[\frac{d_{t+j}^S}{\prod_{i=0}^{j}(1+R_{t+i})}\right].$$

Assume constant dividend and interest rate

$$\Rightarrow p^S = \frac{d^S}{1+R}\left(1 + \frac{1}{1+R} + \frac{1}{(1+R)^2} + \frac{1}{(1+R)^3} + \ldots\right),$$

$$p^S = \frac{d^S}{1+R}\left(\frac{1}{1-\frac{1}{1+R}}\right) = \frac{d^S}{R}.$$

$$R = \frac{d^S}{p^S}.$$

Balanced Exogenous Growth Path, No Uncertainty

$$
\begin{aligned}
d_t^S &= A_{Gt} l_t^{\gamma} k_t^{1-\gamma} - w_t l_t - i_t, \\
\frac{d_{t+1}^S}{d_t^S} &= \frac{y_{t+1} - w_{t+1} l_{t+1} - i_{t+1}}{y_t - w_t l_t - i_t} \\
&= \frac{y_t(1+g) - w_t(1+g) l_t - i_t(1+g)}{y_t - w_t l_t - i_t} = 1 + g.
\end{aligned}
$$

Balanced Exogenous Growth Path, No Uncertainty

$$p_t^S = \frac{d_t^S}{1+R} + \frac{d_t^S(1+g)}{(1+R)^2} + \frac{d_t^S(1+g)(1+g)}{(1+R)^3} + \ldots,$$

$$p_t^S = \frac{d_t^S}{1+R}\left(\frac{1}{1-\frac{(1+g)}{(1+R)}}\right) = \frac{d_t^S}{1+R}\left(\frac{1+R}{(1+R)-(1+g)}\right),$$

$$p_t^S = \frac{d_t^S}{R-g}.$$

Defining risk as covariance

$$X_t \equiv \frac{1}{1+R_{t+1}},\ Y_t \equiv p_{t+1}^S + d_{t+1}^S.$$

$$p_t^S = E_t[X_t Y_t] = E_t[X_t]E_t[Y_t] + Cov_t[X_t Y_t].$$

Suppose $Cov_t[x_t, y_t] \neq 0$,

$$p_t^S = E_t\left[\frac{\beta \frac{\partial u(c_{t+1}, x_{t+1})}{\partial c_{t+1}}}{\frac{\partial u(c_t, x_t)}{\partial c_t}} (p_{t+1}^s + d_{t+1}^s) \right]$$

$$1 = \frac{1}{(1+R_{t+1})} \frac{E_t[p_{t+1}^S + d_{t+1}^S]}{p_t^S}$$
$$+ Cov_t\left[\left(\frac{\beta \frac{\partial u(c_{t+1}, x_{t+1})}{\partial c_{t+1}}}{\frac{\partial u(c_t, x_t)}{\partial c_t}} \right) \left(\frac{p_{t+1}^S + d_{t+1}^S}{p_t^S} \right) \right]$$

And with a definition and a little arithmetic

Definition: $$1 + E_t[R^S_{t+1}] \equiv E_t\left(\frac{p^S_{t+1} + d^S_{t+1}}{p^S_t}\right)$$

$$E_t[R^S_{t+1}] - R_{t+1} = -(1 + R_{t+1})Cov_t\left[\frac{\beta\frac{\partial u(c_{t+1}, x_{t+1})}{\partial c_{t+1}}}{\frac{\partial u(c_t, x_t)}{\partial c_t}}, 1 + R^S_{t+1}\right]$$

And log utility

$$E_t[R^S_{t+1}] - R_{t+1} = -\left(\frac{1 + R_{t+1}}{1 + \rho}\right)Cov_t\left[\frac{c_t}{c_{t+1}}, 1 + R^S_{t+1}\right],$$

$$R_{t+1} \simeq \rho;$$

$$E_t[R^S_{t+1}] - R_{t+1} = -Cov_t\left[\frac{c_t}{c_{t+1}}, 1 + R^S_{t+1}\right].$$

Equity Premium Puzzle

- Covariance of consumption growth and stock returns is about -.006: predicts EP = 0.6%.
- Equity returns exceed risk free bond returns by 6%.
- The equation is off by an order of magnitude.
- Explain how adding human capital to the model might explain the gap.

Homework for May 1

- Read Chapter 18. Public Finance pages 766-791
- Quiz 23

1. Write out the Government's Wealth constraint.
2. What is Ricardian Equivalence?
3. What is the transversality condition that underlies Ricardian equivalence?
4. Explain the advantages and disadvantages of printing money rather than raising taxes to pay for government spending.