Chapters 17: Asset Prices and Finance (pp. 730-762)

April 24, 2017

Homework for April 24

- Finish Growth Homework (Chapter 11 and 12) and Quizzes 19 and 20.
- Read Chapter 17
- Quiz 21

1. Set up and solve the model with bonds (Sections 17.2.1 and 17.2.2
2. Set up the model in which the household owns stock in the firm rather than renting capital to the firm.
3. Derive the value of the firm using the firm's Bellman equation.
4. What real world problems are solved by having equity financing with firms owning capital rather than households?

Start with the baseline model of Chapter 8

$$y_t = A_G l_t^{\gamma} k_t^{1-\gamma} = c_t + i_t,$$

$$i_t = k_{t+1} - (1 - \delta_k) k_t,$$

$$1 = x_t + l_t,$$

$$c_t = w_t l_t + r_t k_t - k_{t+1} + k_t (1 - \delta_k).$$

$$V(k_t) = \underset{c_t, x_t, l_t, k_{t+1}}{Max} : \ln c_t + \alpha \ln x_t + \beta V(k_{t+1}),$$

$$V(k_t) =$$

$$\underset{l_t, k_{t+1}}{Max} : \ln[w_t l_t + r_t k_t - k_{t+1} + k_t (1 - \delta_k)] + a \ln(1 - l_t) + \beta V(k_{t+1}).$$

Add savings in Government Bonds to the household budget constraint.

$$G_t = b_{t+1} - b_t (1 + R_t).$$

$$c_t = w_t l_t + r_t k_t - k_{t+1} + k_t(1-\delta_k) + G_t - b_{t+1} + b_t(1+R_t).$$

$$V(k_t, b_t)$$

$$= \underset{l_t, k_{t+1}, b_{t+1}}{Max} \ln[w_t l_t + G_t - k_{t+1} + k_t(1 + r_t - \delta_k) - b_{t+1} + b_t(1+R_t)]$$

$$+ \alpha \ln(1 - l_t) + \beta V(k_{t+1}, b_{t+1});$$

$$0 = \frac{1}{c_t} w_t + \frac{\alpha}{x_t}(-1), \ \frac{1}{c_t}(-1) + \beta \frac{\partial V(k_{t+1}, b_{t+1})}{\partial k_{t+1}},$$

FOCs

$$\frac{\partial V(k_t, b_t)}{\partial k_t} = \frac{\partial u(c_t, x_t)}{\partial c_t}(1 + r_t - \delta_k),$$

Envelope Condition

Equilibrium Conditions at Margin

$$\beta \equiv \frac{1}{1+\rho}, \quad \Rightarrow \frac{c_t}{c_{t-1}} = \frac{1 + r_t - \delta_k}{1+\rho}, \ w_t = \frac{\frac{\alpha}{x_t}}{\frac{1}{c_t}};$$

$$r_t = (1-\gamma) A_G l_t^{\gamma} k_t^{-\gamma}, \ w_t = \gamma A_G l_t^{\gamma-1} k_t^{1-\gamma}.$$

New FOC and Envelope Condition with Bonds

New FOC and Envelope Condition with Bonds

$$b_{t+1} \quad : \quad 0 = \frac{\partial u(c_t, x_t)}{\partial c_t}(-1) + \beta \frac{\partial V(k_{t+1}, b_{t+1})}{\partial b_{t+1}},$$

$$b_t \quad : \quad \frac{\partial V(k_t, b_t)}{\partial b_t} = \frac{\partial u(c_t, x_t)}{\partial c_t}(1 + R_t),$$

$$\Rightarrow 0 = \frac{\partial u(c_{t-1}, x_{t-1})}{\partial c_{t-1}}(-1) + \beta \frac{\partial V(k_t, b_t)}{\partial b_t},$$

$$\frac{\partial V(k_t, b_t)}{\partial b_t} = \frac{\partial u(c_{t-1}, x_{t-1})}{\beta \partial c_{t-1}}, \quad \frac{\frac{\partial u(c_{t-1}, x_{t-1})}{\partial c_{t-1}}}{\beta \frac{\partial u(c_t, x_t)}{\partial c_t}} = 1 + R_t;$$

$$\frac{c_t}{\beta c_{t-1}} = 1 + R_t, \;\; \beta \equiv \frac{1}{1+\rho}, \;\; \frac{c_t}{c_{t-1}} = \frac{1+R_t}{1+\rho};$$

$$\frac{c_t}{c_{t-1}} = \frac{1 + r_t - \delta_k}{1+\rho}$$

$$\Rightarrow R_t = r_t - \delta_k.$$

Equity Capital Asset Market

- If returns are certain and legal treatment is the same, firm's ownership shares yield same as bonds.
- But, in reality, bonds are treated better in bankruptcy and the return is far from certain. With such risk, firm's dividend payout on equity shares is higher than the return on risk-free bonds.
- Set up the model with certain returns to stock rather than direct returns to owning capital.

$$+ r_t k_t - k_{t+1} + k_t(1-\delta_k)$$

$$c_t = w_t l_t + G_t - b_{t+1} + b_t(1+R_t) - p_t^S s_{t+1} + (p_t^S + d_t^S) s_t,$$

Baseline Model with investment in Equity

- Some notation:

Denote share of goods producer by s_t, with price p_t^S;
time t net investment: $-p_t^S s_{t+1} + p_t^S s_t$; dividends $d_t^S s_t$:

$$c_t = w_t l_t + G_t - b_{t+1} + b_t(1 + R_t) - p_t^S s_{t+1} + (p_t^S + d_t^S)s_t,$$

$$V(b_t, s_t) = \underset{l_t, b_{t+1}, s_{t+1}}{Max}$$

$$\ln[w_t l_t + G_t - b_{t+1} + b_t + b_t R_t - p_t^S s_{t+1} + (p_t^S + d_t^S)s_t]$$

$$+ \alpha \ln(1 - l_t) + \beta V(b_{t+1}, s_{t+1});$$

FOC, EC, and new equilibrium conditions

$$s_{t+1} \;:\; 0 = \frac{\partial u(c_t, x_t)}{\partial c_t}(-p_t^S) + \beta \frac{\partial V(b_{t+1}, s_{t+1})}{\partial s_{t+1}},$$

$$s_t \;:\; \frac{\partial V(b_t, s_t)}{\partial s_t} = \frac{\partial u(c_t, x_t)}{\partial c_t}(p_t^s + d_t^s);$$

$$\Rightarrow 1 + R_t = \frac{\partial[u(c_{t-1}, x_{t-1})]/\partial c_{t-1}}{\beta[\partial u(c_t, x_t)/\partial c_t]} = \frac{p_t^S + d_t^S}{p_{t-1}^S} \equiv 1 + R_t^s.$$

Firm’s Goal is to Maximize Stockholder Wealth

$$s_t d_t^S = A_G l_t^{\gamma} k_t^{1-\gamma} - w_t l_t - k_{t+1} + k_t(1-\delta_k) + p_t^S s_{t+1} - p_t^s s_t.$$

$$s_t = s_{t+1} = 1.$$

$$p_t^S s_{t+1} - p_t^S s_t = p_t^S - p_t^S = 0.$$

$$d_t^S = A_G l_t^{\gamma} k_t^{1-\gamma} - w_t l_t - k_{t+1} + k_t(1-\delta_k).$$

Bellman Equation for the Firm Problem

$$p^S(k_t) = \underset{l_t, k_{t+1}}{Max} \left(d_t^S + \left(\frac{\beta \frac{\partial u(c_{t+1}, x_{t+1})}{\partial c_{t+1}}}{\frac{\partial u(c_t, x_t)}{\partial c_t}} \right) p^S(k_{t+1}) \right)$$

$$= \underset{l_t, k_{t+1}}{Max} \left(d_t^S + \frac{p^S(k_{t+1})}{1 + R_{t+1}} \right)$$

$$= \underset{l_t, k_{t+1}}{Max} \left(A l_t^{\gamma} k_t^{1-\gamma} - w_t l_t - k_{t+1} + k_t(1 - \delta_k) + \frac{p^S(k_{t+1})}{1 + R_{t+1}} \right)$$

Compute the FOCs, the Envelope Condition and the Equibrium Conditions

FOC, EC

$$\frac{\partial p^s(k_t)}{\partial l_t} = \left(\gamma A l_t^{\gamma-1} k_t^{1-\gamma}\right) - w_t = 0$$

$$\frac{\partial p^s(k_t)}{\partial k_{t+1}} = (-1) + \frac{1}{1+R_{t+1}} \frac{\partial p^s(k_{t+1})}{\partial k_{t+1}} = 0$$

$$\frac{\partial p^s(k_t)}{\partial k_t} = (1-\gamma) A l_t^{\gamma} k_t^{-\gamma} + (1-\delta_k)$$

Equilibrium Conditions

$$R_t = (1-\gamma) A_G \left(\frac{l_t}{k_t}\right)^{\gamma} - \delta_k,$$

$$r_t \equiv (1-\gamma) A_G \left(\frac{l_t}{k_t}\right)^{\gamma},\ R_t = r_t - \delta_k;\ w_t = \gamma A_G \left(\frac{l_t}{k_t}\right)^{\gamma-1}.$$

Valuing Equity

- What is the stock worth to the household?
- Firm manager maximizes the shareholder's value.
- What is the firm worth?
- What is the present value of the firm?

Value of firm is value of capital

$$p^S(k_t) = A_G l_t^{\gamma} k_t^{1-\gamma} - w_t l_t - k_{t+1} + k_t(1-\delta_k) + \frac{p^S(k_{t+1})}{1+R_{t+1}},$$

$$y_t = A_G l_t^{\gamma} k_t^{1-\gamma} = w_t l_t + r_t k_t,$$

$$p^S(k_t) = w_t l_t + r_t k_t - w_t l_t - k_{t+1} + k_t(1-\delta_k) + \frac{p^S(k_{t+1})}{1+R_{t+1}},$$

$$p^S(k_t) = k_t(1 + r_t - \delta_k) - k_{t+1} + \frac{p^S(k_{t+1})}{1+R_{t+1}}.$$

Subsitute forward in the Bellman equation

Remember the value function for utility was the maximum present value of lifetime utility.

$$p^S(k_t) = A_G l_t^\gamma k_t^{1-\gamma} - w_t l_t - k_{t+1} + k_t(1-\delta_k) + \frac{p^S(k_{t+1})}{1+R_{t+1}},$$

$$y_t = A_G l_t^\gamma k_t^{1-\gamma} = w_t l_t + r_t k_t,$$

$$p^S(k_t) = w_t l_t + r_t k_t - w_t l_t - k_{t+1} + k_t(1-\delta_k) + \frac{p^S(k_{t+1})}{1+R_{t+1}},$$

$$p^S(k_t) = k_t(1 + r_t - \delta_k) - k_{t+1} + \frac{p^S(k_{t+1})}{1+R_{t+1}}.$$

Here the Maximum is the present value of lifetime dividends

$$p^S(k_t) = k_t(1 + r_t - \delta_k) - k_{t+1} + \frac{k_{t+1}(1 + r_{t+1} - \delta_k) - k_{t+2} + \frac{p^S(k_{t+2})}{1+R_{t+2}}}{1 + R_{t+1}},$$

$$p^S(k_t) = k_t(1 + r_t - \delta_k) - k_{t+1} + \frac{k_{t+1}(1 + r_{t+1} - \delta_k) - k_{t+2} + \frac{p^S(k_{t+2})}{1+R_{t+2}}}{1 + R_{t+1}},$$

$$= k_t(1 + r_t - \delta_k) - k_{t+1} + k_{t+1} - \frac{k_{t+2}}{1 + R_{t+1}} + \frac{p^S(k_{t+2})}{(1 + R_{t+1})(1 + R_{t+2})}$$

$$= k_t(1 + r_t - \delta_k) - \frac{k_{t+2}}{1 + R_{t+1}} + \frac{p^S(k_{t+2})}{(1 + R_{t+1})(1 + R_{t+2})}.$$

Here the Maximum is the present value of lifetime dividends

$$p^S(k_t) = k_t(1 + r_t - \delta_k)$$

$$+ \lim_{j\to\infty} \frac{p^S(k_j)}{(1+R_{t+1})(1+R_{t+2}) \bullet \bullet \bullet (1+R_{t+j})};$$

$$0 = \lim_{j\to\infty} \frac{p^S(k_j)}{(1+R_{t+1})(1+R_{t+2}) \bullet \bullet \bullet (1+R_{t+j})};$$

$$p^S(k_t) = k_t(1 + r_t - \delta_k)$$

Homework for April 26

- Finish reading chapter 17, from section 17.4 to the end (pages743-760)
- Quiz 22

1. How does the introduction of uncertainty complicate our solutions?
2. What is the Equity Premium?
3. What is the Expectations Hypothesis of Asset Prices (Section 17.4.2)?
4. Explain in words what you think is the Expectations Hypothesis of Interest Rates.